68th Annual Congress of the South African Mathematical Society:

Book of Abstracts

2 - 5 December 2025,

Nelson Mandela University, Gqeberha

Information

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Welcome message to the 68th Annual Congress of the South African Mathematical Society (SAMS)

Dear Esteemed Delegate,

The advancement of mathematics in South Africa is the key objective of SAMS, which seeks to promote all disciplines of mathematics to the high standards of research, teaching and learning. On behalf of the organizing committee, it a great pleasure to welcome you to the 68th Annual Congress of SAMS.

Over the next few days, you will have opportunities to explore the latest developments in your field of expertise by attending lectures, engage with colleagues through stimulating discussions, build new contacts by networking with peers.

We are grateful to the the council of SAMS for advice and guidance during the planning phase of the conference. We also acknowledge financial support from the Faculty of Science of Nelson Mandela University, the South African Mathematical Society (SAMS), the DST / NRF Centre for Excellence of Mathematical and Statistical Sciences (CoE-MaSS) and the National Institute for Theoretical and Computational Sciences of Southern Africa (NITheCS).

A very warm welcome and we hope that you will enjoy the congress.

The organizing committee

Martin Weigt

Contents

1 Invited speakers

The Oriented Diameter of a Graph

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SAMS Special Session: Invited Talk
SAMS Classification: 6

Assume that we wish to turn all streets of a city into one-way streets. We wish to do that in a way such that it is still possible to travel between any two locations in the city, while keeping possible increases in travel distances between locations as small as possible. This problem can be modeled by graphs.

An orientation of a graph G is a digraph that is obtained from G by assigning a direction to every edge. The diameter of a graph or digraph is the largest among the distances between its vertices. The oriented diameter of G is defined as the minimum diameter among all strong orientations of G, i.e., orientations in which there is a path between any two vertices in either direction. The study of the oriented diameter was initiated in the classical paper [1] by Chvátal and Thomassen. In this talk we present a selection of results in this area of research and report on exciting new developments.

References:

(1) V. Chvátal, C. Thomassen, *Distances in orientations of graphs*, Journal of Combinatorial Theory Series B, **24** (1978) 61–75.

Recent developments in semi-abelian categorical algebra

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SAMS Special Session: Invited Talk
SAMS Classification: 4

The inception of categorical algebra was soon after that of category theory itself and can essentially be traced to (11). The initial development of categorical algebra was largely dominated by abelian categories, although there was also a successful non-abelian part. After this initial wave the development of category theory was largely dominated by topos theory. With the general development of category theory, including the introduction of Barr-exact categories (1), categorical algebra development had a resurgence starting in 1990s. This development included Mal'tsev categories in (6), Bourn-protomodularity in (4), and semi-abelian categories in (10), which also linked the early non-abelian development with this newer development.

This talk is about interesting additional conditions on a semi-abelian category that hold for the categories of groups, Lie algebras, (non-unital) associative algebras over a commutative ring, and more generally for Orzech's categories of interest (12), but not for say the category of non-associative algebras (which is semi-abelian), which have been introduced by various authors. Roughly speaking, categories of interest are varieties of universal algebras, whose algebras have a group structure together with binary operations that distribute over the group operation and which satisfy a *generalized associativity law*. We explain that (surprisingly) there are two conditions that imply all other conditions (known to the author) that hold for all categories of interest. These conditions are the existence of all action cores (3) and the existence of all normalizers (9). In particular, we will explain how these two conditions give rise to fiberwise-algebraic-cartesian-closedness (8), action accessibility (5), algebraic coherence (7), and hence various commutator coincidence conditions as well as strong protomodularity (2).

The remainder of the talk will be devoted to presenting recent examples of categories which are not categories of interest but nevertheless have action cores or normalizers.

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(5) D. Bourn and G. Janelidze, *Centralizers in action accessible categories*, Cahiers de Topologie et Géométrie Différentielles Catégoriques **50**(3), 211–232, 2009.

(6) A. Carboni, J. Lambek, and M. C. Pedicchio, *Diagram chasing in Mal'ccev categories*, Journal of Pure and Applied Algebra **69**(3), 271–284, 1991.

(7) A. S. Cigoli, J. R. A. Gray, and T. Van der Linden, *Algebraically coherent categories*, Theory and Applications of Categories **30**(54), 1864–1905, 2015.

(8) J. R. A. Gray, *Algebraic exponentiation and internal homology in general categories*, Ph.D. thesis, University of Cape Town, 2010.

(9) J. R. A. Gray, *Normalizers, centralizers and action representability in semi-abelian categories*, Applied Categorical Structures **22**(5-6), 981–1007, 2014.

(10) G. Janelidze, L. Márki, and W. Tholen, *Semi-abelian categories*, Journal of Pure and Applied Algebra **168**, 367–386, 2002.

(11) S. Mac Lane, *Duality for groups*, Bulletin of the American Mathematical Society **56**(6), 485–516, 1950.

(12) G. Orzech, *Obstruction theory in algebraic categories I, II*, Journal of Pure and Applied Algebra **2**(4), 287–314 and 315–340, 1972.

Nonstandard Finite Difference Schemes in Epidemiology: The case of Ebola Virus and HIV-Mpox diseases

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SAMS Special Session: Invited Talk
SAMS Classification: 5, 10

Dynamical systems play a vital role in epidemiology modelling. However, most of the differential equations cannot be completely solved by analytical techniques. Consequently, reliable numerical methods and simulations are of fundamental importance in gaining insight into the solutions of the system under consideration. In this talk, we adopt the Nonstandard Finite Difference (NSFD) Method, created by R. Mickens in the late eighties. Guided by the construction of exact schemes of simple models and Mickens'rules, we first formalize the definition of the NSFD method, and discuss its methodology using various techniques such as sub-equations, conservation law and destructive-constructive structure of the continuous model. We next investigate two models for the Ebola virus transmission and the HIV influence on the spread of the Mpox disease. We finally construct NSFD schemes that preserve the rich dynamics of the continuous models such as the positivity & boundedness of solutions, the stability of disease-free & endemic equilibria, and the conservation laws.

References:

(1) R. Anguelov and J. Lubuma, *Contribution to the mathematics of the nonstandard finite difference method and applications*, Numerical Methods for Partial Differential Equations, **17** (2001) 518–543.

(2) R. Anguelov, T. Berge, M, Chapwanya, J.K. Djoko, P. Kama, J. Lubuma and T. Terefe, *Nonstandard finite difference method revisited and application to the Ebola Virus Disease transmission dynamics*, Journal of Difference Equations and Applications, **26** (2020) 818–854.

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(4) A.J. Ouemba Tassé, Y. Terefe and J. Lubuma, *Assessing the influence of HIV on the spread of Mpox disease,* Mathematical Biosciences, **387** (2025) 109499.

(5) K.C. Patidar, *Nonstandard finite difference methods: recent trends and further developments,* Journal of Difference Equations and Applications, **22** (2016) 817–849.

Mathematical principles of symmetry breaking and pattern formation

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SAMS Special Session: Invited Talk
SAMS Classification: 10

Cells and tissues are physical objects: despite their molecular complexity, they obey physical and chemical laws. This raises a central question for mathematical biology: which mathematical principles enable complex biological patterns to arise from simple, well-defined dynamical rules? In this talk, I will present a mechanochemical, PDE-based framework motivated by recent experiments on regenerating epithelia, with Hydra as a model system. The model couples reaction–diffusion dynamics of a morphogen with continuum mechanical equations for tissue deformation. A positive feedback between mechanical strain and morphogen production leads to symmetry breaking and the emergence of stable single-peaked steady states. Using bifurcation analysis, linear stability theory, and asymptotic arguments, I will characterise the onset of patterning and identify the parameter regimes in which these solutions exist and are robust. This mechanism provides a mathematically distinct route to effective long-range inhibition, unlike curvature–morphogen coupling or classical Turing-type diffusion-driven instabilities. Model predictions show quantitative agreement with experimental data, suggesting that mechanically mediated feedback may constitute a fundamental organising principle in epithelial pattern formation.

Compact disk and sphere packings

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Compact disk and sphere packings are interesting geometric/combinatorial structures. These structures have been observed "in the wild", and for this reason they also attract some interest from outside of mathematics.

We will discuss some of recent history surrounding the study of compact packings. Along the way, we will point out some of the mathematical and computational problems that arise in studying compact packings, and how these problems connect the subject to many other subfields of mathematics.

SAMS Special Session: Mid-career plenary speaker
SAMS Classification: 12

Leading a new pedagogical approach in mathematics teaching: A case for mathematical modelling education in South Africa

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SAMS Special Session: Invited Talk (AMESA)
SAMS Classification: 16

The theme of my presentation at this 68th South African Mathematics Congress (SAMS) 2025 is about integrating mathematical modelling into the national mathematics curricula – which has increasingly highlighted the importance of students engaging productively in complex and often unfamiliar model-eliciting activities (MEAs). Mathematical modelling serves as a "didactical vehicle for developing modelling competency and enhancing students' conceptual learning [and understanding] of mathematics" (1). During the past four decades, many countries have integrated mathematical modelling as teaching strategy into the teaching and learning of mathematics in schools and higher education institutes (3). In South Africa, the curriculum emphasizes "mathematical modelling is an important focal point. Real-life problems should be incorporated into all sections whenever appropriate" (2). A systematic review of mathematical modelling education in South Africa between 2003 and 2022 showed that research in this area is increasing (4). However, there is limited practical integration of mathematical modelling in classrooms, because it is considered difficult by both teachers and students. Furthermore, the rigid high-stakes national examination does not assess mathematical modelling competencies. Building on this context, the presentation argues that integrating mathematical modelling into high school classrooms is not only essential but also challenging, as students often encounter productive struggles when engaging with MEAs. To address these challenges, the presentation illustrates—through concrete examples—how key design principles of MEAs can guide teachers in transforming conventional textbook problems into rich modelling tasks that support students learning.

References:
(1) M. Blomhoj and T.H. Kjeldsen, *Students mathematical learning in modelling activities*, In G. A. Stillman, G. Kaiser, W. Blum, J. Brown (Eds.),

Teaching mathematical modelling: Connecting to research and practice. International perspectives on the teaching and learning of mathematical modelling (pp. 141–151), 2023, Springer, Dordrecht.

(2) Department of Basic Education. National Curriculum Statement. Mathematics FET Curriculum and Assessment Policy Statement. Government Printers. Pretoria, 2011.

(3) J. Huang, X. Lu and B. Xu, *The historical development of mathematical modelling in curricular standards/syllabi in China* In F. K. S. Leung, G. A. Stillman, G. Kaiser, and K. L. Wong (Eds.), Mathematical modelling education in east and west (pp. 177–188), 2021, Springer International Publishing.

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Finite flat group schemes over $\mathbb{Z}$

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SAMS Special Session: Invited Talk
SAMS Classification: 1

Finite flat group schemes over rings of integers play an important role in number theory. They are built up out of simple group schemes. In 1966 John Tate asked whether simple finite flat group schemes over $\mathbb{Z}$ necessarily have prime order.

In this talk we put this problem in a context and answer Tate's question in a very special case.

On Generalized B*-Algebras

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The study of bounded operator algebras began in the late 1930s to early 10940s with the introduction of rings of operators (now called von Neumann algebras) through a series of ground breaking papers by John von Neumann and his collaborators F.J. Murray. At around 1943, I.M. Gelfand and M.A. Neumark introduced C*-algebras. Since these times, von Neumann algebras and C*-algebras have exploded into a rich field of study which also influenced group representation theory, mathematical physics and K-Theory, to name just a few. One aspect that von Neumann algebras and C*-algebras have in common is that they can be identified with $*$-algebras of bounded linear operators on a Hilbert space, i.e., they are bounded operator algebras.

Observables in quantum mechanics mainly require observables to be represented as unbounded linear operators on a Hilbert space. Adding to this, the late American physicist A.S. Wightman formulated some axioms for relativistic quantum field theory, which, in 1962, were reformulated by A. Uhlmann in terms of unbounded operators. This, along with group representation theory, provided motivation for a general investigation of $*$-algebras consisting of unbounded linear operators on a Hilbert space (these are referred to as unbounded operator algebras for short). Unbounded operator algebras are not as well developed as bounded operator algebras mentioned above, and are more complicated to work with. For example, the concept of commutant or tensor product in bounded operator algebras have more than one generalization in unbounded operator algebras. Only a small handful of mathematicians are working in unbounded operator algebras, compared to bounded operator algebras. Much of what is currently known about unbounded operator algebras are captured in (1), (4) and (6).

This talk will focus mainly on a special class of unbounded operator algebras, namely, generalized B*-algebras (GB*-algebras for short). These are locally convex $*$-algebras which are generalizations of C*-algebras. They were first introduced by G.R. Allan in (1), and developed further by P.G. Dixon

in (3) to include non-locally convex algebras. Much of the work on GB*-algebras is scattered across the literature, and the author and his collaborators recently published a monograph on this topic in (5). Since GB*-algebras are not normed, and unbounded operators are involved, one does not have as many techniques available as in the case of C*-algebras and von Neumann algebras. The aim of this talk is to give a general overview of GB*-algebras, along with how the author and his collaborators have contributed to the field.

References:

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(2) G.R Allan, *On a class of locally convex algebras*, Proc. London. Math. Soc., **17** (1967) 91 – 114.

(3) P. G. Dixon, *Generalized B*-algebras*, Proc. London Math. Soc., **21** (1970) 693 – 715.

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2 Special Session: Categories, Algebra, Topology and Logic

Session organisers: Themba Dube (Dubeta@unisa.ac.za, Amartya Goswami (agoswami@uj.ac.za)

On extensions of Cohen Structure Theorem

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 7

We will investigate for which rings R there exists a maximal ideal $\mathcal{M}$ of R such that the canonical projection $R \to R/\mathcal{M}$ has a section, so that $R/\mathcal{M}$ is isomorphic to a field K contained in R. Cohen Structure Theorem (1) ensures that this is true for any equicharacteristic complete local Noetherian commutative ring. In this talk, we will present two equivalent characterizations of this property and use them to exhibit two classes of rings that satisfy it. Moreover, we will provide several examples (not necessarily local or complete local), as well as methods to construct new examples. The talk is based on the preprint (2).

References:
(1) I. S. Cohen, On the structure and ideal theory of complete local rings, Trans. Amer. Math. Soc., 59 (1946), 54–106.
(2) E. Caviglia, A. Goswami and L. Mesiti, On extensions of Cohen Structure Theorem, arXiv preprint 2502.09562 (2025).

Dual spaces of lattices and semidistributive lattices

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 34

Representation and duality theorems play an important role in the study of lattice-based algebras. Stone duality for Boolean algebras and Priestley duality for distributive lattices are two well-known examples. Building on the work of Ploščica [3], we describe a class of dual spaces for bounded lattices, with the lattices being represented via digraphs with topology. Then, using a result from Jónsson and Rival [2], we extend our previous representation results on semidistributive lattices [1] from the finite case to arbitrary semidistributive lattices. This representation is obtained by introducing an additional digraph condition on the so-called Ploščica spaces.

References:

[1] A Craig, M Haviar, J São João, *Dual digraphs of finite semidistributive lattices*, CUBO **24** (2022) 369–392.

[2] B Jónsson, I Rival, *Lattice varieties covering the smallest non-modular variety*, Pacific. J. Math. **82** (1979) 463–478.

[3] M Ploščica, *A natural representation of bounded lattices*, Tatra Mountains Math. Publ. **5** (1995) 75–88.

Monilmorphisms

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 4

We introduce a new class of morphisms in a pointed category with kernels, which contains all monomorphisms and all null morphisms. We call a morphism $f\colon X \to Y$ in such a category a *monilmorphism* when for any two morphisms $g_1, g_2\colon W \to X$ we have:

$$[f \circ g_1 = f \circ g_2 \,\wedge\, \mathsf{ker}(f \circ g_1) = \mathsf{ker}(g_1) \,\wedge\, \mathsf{ker}(f \circ g_2) = \mathsf{ker}(g_2)] \implies [g_1 = g_2].$$

In the category of sets and partial functions, monilmorphisms are precisely the injective partial functions. We generalise this fact to a restriction category (1) in that we describe when monilmorphisms coincide with restricted monics here.

We establish general properties of monilmorphisms and characterize them in various types of categories. In particular, we show that while every monilmorphism has the property that subobjects of its codomain can have at most one cross-section over it, this property characterises monilmorphisms in any normal category in the sense of (2). In a number of particular normal categories (e.g. $\mathbf{Set}_*^{\mathrm{op}}$ and $\mathbf{Vect}_K$) monilmorhpisms reduce to just monos and zero morphisms. We show on the other hand that if all coequalisers exist in addition, every morphism from an object with 0-distributive subobject lattice is a monilmorphism.

References:

(1) J.R.B. Cockett and S. Lack, *Restriction categories I: categories of partial maps*, Theoretical Computer Science, **270** (2002) 223–259.
(2) Z. Janelidze, *The pointed subobject functor, 3×3 lemmas, and subtractivity of spans*, Theory and Applications of Categories, **23** (2010) 221-–242.

Extending Cohen Structure Theorem

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 7

In this talk, we discuss extensions of the Cohen Structure Theorem beyond the setting of local rings. We shall show that both Cohen Structure Theorem and Nagata's generalization of it arise as special cases of our results.
Note. This is joint work with Elena Caviglia and Luca Mesiti from Stellenbosch University, South Africa.

Weil's Entourages and Order Relations In Point-free Topology

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

In his Ph.D. thesis (1), J. Picado introduced the so-called Weil's entourages in point-free topology. Among many other findings, he proved that both the categories of Weil's uniformities and Weil's quasi-uniformities are order-isomorphic to the categories of covering uniformities and covering quasi-uniformities (2), respectively. He further introduced the category of Weil's nearness. However, due to the lack of certain properties, he was unable to establish an isomorphism between this category and the category of covering nearness [2]. In a recent work (3), we established a functorial correspondence between the category of covering pre-nearness and that of semi-Császár structures. In this talk, we aim to relate the category of Weil's pre-uniformities, which we have introduced, to that of semi-Császár structures. Using Császár orders (4), we will show that, under suitable restrictions, covering nearness and Weil's nearness coincide in a specific way. Our method of proof follows closely the approach used in (3).

References:

(1) J Picado, *Weil entourages in pointfree topology, Ph.D.* Thesis, University of Coimbra, 1995.

(2) J Picado and P. *Aleš, Frames and Locales: topology without points.* Springer Science & Business Media, 2011.

(3) D Holgate and Bakulikira C. Iragi *Császár structures and pre-nearness on frames.* (Under review), 2025.

(4) A Császár, *Foundations of general topology*, Pergamon, 1963.

Kernels for commutative hemirings and subtractive varieties

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 35, 29, 4

A *commutative hemiring* is a commutative ring without the requirements of existence of identity and additive inverses (but with the law $0x = 0$). In this talk we give a characterization of kernels in the category of commutative hemirings, inspired by the their characterization as subtractive ideals, and show that this characterization holds for all subtractive varieties of universal algebras as well, even though the category of commutative hemirings is not a subtractive variety. The question then is: what common properties of subtractive varieties and commutative hemirings ensure such characterization?

Normalizers of Heyting Semilattices

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 4, 34, 35

A normalizer of a subgroup H of a group G can be described as the largest subgroup $N_G(H)$ of G such that H is normal in $N_G(H)$. The formula for $N_G(H)$ is given as

$$N_G(H) = \{g \in G \mid gHg^{-1} = H\}.$$

The notion of a normalizer has been formulated in any pointed category (see [1]), and has been studied in other cases, such as Lie Algebras. This talk explores normalizers of Heyting semilattices.

A Heyting semilattice is a meet-semilattice X with a top element 1 that is equipped with a binary operation $\rightarrow$ satisfying

$$x \wedge y \leq z \iff x \leq y \rightarrow z.$$

A subalgebra of a Heyting semilattice X is normal if and only if it is the kernel of some homomorphism $f : X \rightarrow Y$. A normalizer of a subalgebra S of a Heyting semilattice X (if it exists) is defined as the largest subalgebra $N_X(S)$ such that S is normal in $N_X(S)$.

In this talk, it is shown that the subvariety of Heyting semilattices generated by the chain of length 3 has normalizers, and it is also shown that this is the largest subvariety of Heyting semilattices which has normalizers. Furthermore, a formula for the normalizer (in the case when it does exist) is given.

References:
(1) J.R.A Gray, *Normalizers, Centralizers and Action Representability in Semi-Abelian Categories*, Applied Categorical Structures, **22** (2014) 981–1007.

Unicoherence in Locales

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

We generalize the concept of unicoherence to the context of frames. Unicoherence, originally introduced by Kuratowski, is a connectedness property that is well studied in classical topology and used to detect holes of a space. We extend the notion of unicoherence to locales and we then investigate its properties. In particular, we will show that many of the known characterizations of unicoherence for topological spaces extend to the setting of locales. Some of these characterizations interestingly involve separation properties for locales.

Putting a ring on it: the marriage between subrings of $\mathcal{R}(L)$ and compactifications of L

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

This talk will explore the intricate relationship between certain subrings of pointfree rings of continuous functions and compactifications of frames, with a particular emphasis on those subrings that determine perfect compactifications of frames. We will discuss some structural properties and characterizations of these subrings and the corresponding compactifications they determine.

On the category of almost κ-compact locales.

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

A space X is called an almost κ-compact if for every open ultrafilter $\mathcal{U}$ of X with $\bigcap \mathcal{U} = \emptyset$, there exists a subfamily $\mathcal{V}$ of $\mathcal{U}$ such that $\bigcap \mathcal{V} = \emptyset$, where $|\mathcal{V}| < \kappa$. Bhaumik and Misra introduced almost κ-compact spaces as a generalization of κ-compact spaces for any infinite cardinal κ. In this talk, we introduce the point-free counterpart of this concept. We demonstrate that our definition is conservative and show some properties of almost κ-compact locales, including a characterization of these locales in terms of filters.

References:
(1) S Bhaumik and DN Misra, *A generalization of K-compact spaces*, Czechoslovak Mathematical Journal, 1971, 625–632.

Point-free view of I-regular sequences of $C(X)$

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 2

We investigate the I-regular sequences of $C(X)$ within a point-free setting, emphasing the structure of ideals, regular elements, and the role of cozero elements. We show that the ring $\mathcal{R}L$ does not admit I-regular sequences of length greater than or equal to two. A frame L is characterised as an almost P-frame if and only if the maximum length of I-regular sequences of $\mathcal{R}L$ is zero. Moreover, the maximum length of I-regular sequences of $\mathcal{R}L$ is at most one if and only if the frame L contains at most one non-almost P-point. Finally, we establish that if (f) is a principal z-ideal in $\mathcal{R}L$, then every I-regular sequence in the localisation of $\mathcal{R}L$ has length at most one.

Isomorphisms of sheaves of endomorphisms of locally finitely presented $\mathcal{O}_X$ -modules with involution

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 1

The purpose of this talk is to continue with our investigations on the basic theory of involutions on sheaves of Azumaya algebras. We specifically seek to analyze, in the setting of quasi-coherent $\mathcal{O}_X$-modules, counterparts of classical results pertaining to involutions of the first kind of trivial algebras. For instance, let V and W be faithfully R-modules, and let $\varphi : \mathcal{E}nd_R(V) \overset{\sim}{\to} \mathcal{E}nd_R(W)$ be an R-algebra isomorphism. Then there are invertible R-module L and an isomorphism $\rho : V \otimes L \overset{\sim}{\to} {}_\varphi W$, where ${}_\varphi W$ is the R-module W endowed with the left $\mathcal{E}nd_R(V)$-structure, such that $\varphi(f) = \rho(f \otimes 1)\rho^{-1}$, for any $f \in \mathcal{E}nd_R(V)$. See (2). We have generalized this result as follows: Let $\mathcal{E}$ and $\mathcal{F}$ be locally finitely presented progenerator $\mathcal{O}_X$-modules and let $\varphi : \mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E}) \overset{\sim}{\to} \mathcal{E}nd_{\mathcal{O}_X}(\mathcal{F})$. Then, there are an invertible $\mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E})$-module $\mathcal{L}$ and an isomorphism $\rho : \mathcal{L} \otimes \mathcal{E} \overset{\sim}{\to} {}_\varphi\mathcal{F}$ of $\mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E})$-modules such that, for any section $\Phi \in \mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E})$, $\rho \circ (1 \otimes \Phi) \circ \rho^{-1} = \varphi(\Phi)$. More precisely, $\mathcal{L}^{-1} = \mathcal{H}om_{\mathcal{E}nd_{\mathcal{O}_X}}(\mathcal{E})({}_\varphi\mathcal{F}, \mathcal{E})$, the inverse $\mathcal{E}_{\mathcal{O}_X}$-module $\mathcal{L}$ with respect to the tensor product. As a side result, for any morphism $\varphi : \mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E}) \to \mathcal{E}nd_{\mathcal{O}_X}(\mathcal{F})$ such that $\mathcal{H}om_{\mathcal{E}nd_{\mathcal{O}_X}}(\mathcal{E})({}_\varphi\mathcal{F}, \mathcal{E})$ is locally finitely $\mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E})$-free,

$$(\mathcal{H}om_{\mathcal{E}nd_{\mathcal{O}_X}}(\mathcal{E})({}_\varphi\mathcal{F}, \mathcal{E}))^{-1} = \mathcal{H}om_{\mathcal{E}nd_{\mathcal{O}_X}}(\mathcal{E})(\mathcal{E}, {}_\varphi\mathcal{F}).$$

Moreover, if $(\mathcal{L}_0, \rho_0)$ is another such pair, then $\mathcal{L}_0 \overset{\sim}{\to} \mathcal{L}$, and on the other hand, ρ_0 is obtained by multiplying ρ by a unit section. For *neutral* algebras, that is, Azumaya algebras of the form $\mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E})$, where $\mathcal{E}$ is a locally free $\mathcal{O}_X$-module of finite positive rank, let $\mathcal{E}$ annd $\mathcal{F}$ be locally free $\mathcal{O}_X$ $\mathcal{O}_X$-modules of finite positive rank, and let $\mathcal{A} = \mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E})$, a neutral Azumaya $\mathcal{O}_X$-algebra. Then, there exist an invertible $\mathcal{O}_X$-module $\mathcal{L}$ and an $\mathcal{O}_X$-module isomorphism $\rho : \mathcal{E} \otimes_{\mathcal{O}_X} \mathcal{L} \overset{\sim}{\to} \mathcal{F}$ such that $\varphi(a) = \rho(a \otimes 1)\rho^{-1}$, for any section a of $\mathcal{E}nd_{\mathcal{O}_X}(\mathcal{E})$. Moreover, by letting $\mathcal{A}$ act canonically on $\mathcal{E}$ and on $\mathcal{F}$ through φ, the map $\rho : \mathcal{E} \otimes_{\mathcal{O}_X} \mathcal{L} \overset{\sim}{\to} {}_\varphi\mathcal{F}$ becomes an isomorphism of $\mathcal{A}$-algebras. See (1).

References:

(1) P Gille; E Neher; C Ruether, *Azumaya algebras and obstructions to quadratic pairs over a scheme*, Transactions of the American Mathematical Society **378** (2025), no. 10, 7199–7245.

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On uniformly Menger elements

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

A subset A of a space X is relatively Menger [1] if for every sequence $\{C_n : n \in \mathbb{N}\}$ of open covers of X, there exists, for each n a finite set $J_n \subseteq C_n$ such that $A \subseteq \bigcup_{n \in \mathbb{N}} J_n$. From the notion of relatively Menger subsets, we define and study uniformly Menger elements in the category of nearness frames. An element a of a nearness frame (L, μ) is uniformly Menger if for every sequence $\{C_n : n \in \mathbb{N}\}$ of uniform covers of L, there exists, for each n a finite set $D_n \subseteq C_n$ such that $a \leq \bigcup_{n \in \mathbb{N}} D_n$. These elements are situated between totally bounded elements and pre-Lindelöf elements. We prove that an element a of a nearness frame (L, μ) is uniformly Menger if and only if the quotient $j_{\downarrow a} : (L, \mu) \to (\downarrow a, \mu_{\downarrow a})$ is uniformly M-bounded.

References:
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The variety of finitely representable distributive quasi relation algebras

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SAMS Special Session: Categories, Algebra, Topology and Logic.
SAMS Classification: 34, 35.

Distributive quasi relation algebras (DqRAs) are distributive lattices equipped with a monoid structure and three unary order-reversing operations. DqRAs are a generalisation of relation algebras, which were first developed by Tarski (1) to study algebras of binary relations. We say that a DqRA is representable when it is isomorphic to a certain algebra of binary relations (2). An open question is whether the class of representable DqRAs forms a variety. We know that the class of representable DqRAs is closed under subalgebras and products, so all that remains is to prove closure under homomorphic images. In this talk, we prove that homomorphic images of *finitely representable* DqRAs are themselves finitely representable. We consider a *finitely representable* DqRA and modify its representation in order to represent its homomorphic images. The modification relies on the fact that in the finite case each homomorphic image arises from a special idempotent element.

References:

(1) A Tarski. *On the calculus of relations.* The Journal of Symbolic Logic, **3** (1941) 73–89.
(2) A Craig and C Robinson. *Representable distributive quasi relation algebras.* Algebra Universalis, (2025) 86:12.

One-local retract in modular metrics

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SAMS Classification: 13

In this talk, we consider the concept of one-local retract in the settings of modular metrics. We extend the well-known results on one-local retract in metric point of view to the framework of modular metrics. In particular, we show that any self-map $\psi : X_w \to X_w$ which satisfies the property $w(\lambda, \psi(x), \psi(y)) \leq w(\lambda, x, y)$ for all $x, y, \in X$ and $\lambda > 0$, has at least one fixed point whenever the collection of all q_w-admissible subsets of X_w is both compact and normal.

References:
(1) A.A.N. Abdou, One-local retract and common fixed point in modular metric spaces, Abstr. Appl. Anal. 2013, Art. ID 672069, 8pp. (2) O. Olela Otafudu, On one-local retract in quasi-metric spaces, Topology Proc. **45** (2015) 271–281. (3) V.V. Chistyakov, Modular metric spaces, I: Basic concepts, Nonlinear Anal. **72** (2010) 1 – 14.

Admissibility of Inference Rules

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 19

In this presentation, we discuss the admissibility of inference rules. A rule is admissible in a logic if its addition to the logic preserves the original set of provable formulas. We will show the admissibility of the irreflexivity rule in the case of temporal logic (1). Additionally we discuss the admissibility of rules that enforce temporal frames to be forward and backward discrete (2). We present a sufficient condition for the admissibility of an inference rule, the proof of which uses p-morphisms.

References:
(1) Dov M. Gabbay, Ian Hodkinson, and Mark A. Reynolds. *Temporal Logic: Mathematical Foundations and Computational Aspects.* Clarendon Press, 1994.
(2) Chantel Marais. *An Essay on Branching Time Logics.* PhD thesis, University of Witwatersrand, 2025.

A Study of Continuity and Preorders on Graph Topological Spaces

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

The idea of defining a topology on directed graphs was first introduced by Ahlborn [1], who explored the relationship between graph theory and point-set topology. In particular, we adopt the following definition in [1]: on a directed graph D, a subset of the vertex set is said to be open if there is no directed edge going out of it. This construction yields a unique topology [1], which we call a graph topological space. In this talk, we discuss why a continuous function between such graph topological spaces does not, in general, correspond to the associated directed graph homomorphism. We then examine a case where these two notions are related by considering a certain type of digraph $T(D)$ constructed from D, so that continuity and directed edge-preserving functions are equivalent. Finally, we define a preorder on $T(D)$.

References:

[1] T.J. Ahlborn, *On directed graphs and related topological spaces*, NASA Contractor Report 77019, Kent State University, 1964.

Linear and homogeneous mappings of Near-vector spaces

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 29

Near-vector spaces are generalisations of vector spaces. The study of mappings of near-vector spaces has been introduced by J. André in 1974. We are interested in the linear mappings of near-vector spaces of finite dimension. In this talk, we first review some definitions and background results, and give a complete characterisation of linear mappings of finite-dimensional near-vector spaces as well as a reconstruction problem for these spaces.

Contractions of quasi relation algebras

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 34, 35

Quasi relation algebras (qRAs) were first introduced by Galatos and Jipsen in (2). These algebras can be viewed as generalisations of relation algebras or as certain expansions of residuated lattices. We identify positive symmetric idempotent elements in qRAs and show that they can be used to construct new qRAs called contractions. A contraction inherits many of the properties of the original qRA. In particular, if the original algebra is distributive, then all its contractions are also distributive. This construction generalises a particular case of an analogous construction of a new relation algebra using an equivalence element (3). As in the case of relation algebras, we can show that if the original distributive qRA is representable as an algebra of binary relations, then its contraction is also representable. The notion of a representable *distributive* qRA was defined by Craig and Robinson (1). We also show that if the positive symmetric idempotents of a distributive quasi relation algebra interacts in a specific way with some other element of the algebra, then the algebra is not *finitely* representable. This is a strengthening of a result in (1).

References:
(1) A Craig and C Robinson, *Representable distributive quasi relation algebras*, Algebra Universalis, **86** 2025.
(2) N Galatos and P Jipsen, *Relation algebras as expanded FL-algebras*, Algebra Universalis, **69** (2013) 1–21.
(3) R McKenzie, *The representation of relation algebras*, PhD thesis, University of Colorado, Boulder, 1966.

Local connectedness in Pointfree Topology

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

In this talk we discuss several generalisations of local connectedness in the context of pointfree topology. In particular we discuss almost locally connected frames, $Z-$locally connectedness frames, sum connected frames and quasi locally connected frames. We show that all these frames in a zero-dimensional frame coincide with local connected frames.

Categorically expressible Mal'tev conditions have trivial spectra

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 35, 4

In this talk, the notion of an equivalence invariant variety is introduced. Equivalence invariance is used to formalise what is meant by a Mal'tev condition being categorically expressible. The notion of equivalence invariance is connected to the spectrum of a variety, through the following theorem: A variety is equivalence invariant if and only if its spectrum is trivial, or full. This result is used to answer the question of the existence of Mal'tev conditions which are not equivalent to a categorical property. Such an example includes the existence of group operations satisfying $x^2 = e$. Equivalence invariance for algebraic categories will be briefly discussed, and a theorem will be stated. The universal algebraic case of this theorem is stated as an open problem.

Differentiation Without Limits: A Categorical Approach

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SAMS Special Session: Categories, Algebra, Topology, and Logic
SAMS Classification: 4

The notion of differentiation, from a calculus perspective, is given by the usual limit-definition of a derivative. As various fields in mathematics started to develop, similar structures were found in different settings, all of which possessed the same algebraic properties as the usual notion of a derivative. For example, a derivation in an algebra setting which satisfies the Leibniz rule. As such, a variety of differential structures started to appear in these different settings. In this talk, we discuss a general notion of a derivative from a categorical perspective (1), and in doing so, isolate some axioms of a derivative that are consistent throughout these different settings. Furthermore, the categorical definition of a derivative captures the known notions of derivatives when considered in specific categories, as seen by means of an example.

References:
(1) J.-S. P. Lemay, *Differential algebras in codifferential categories*, Journal of Pure and Applied Algebra, **223** (2019) 4191–4225.

Investigation of almost compactness in the category of locales.

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 13

This talk is based on a study investigating almost compactness in locales. In the localic framework, almost compactness can be traced back to Jan Paseka and B. Smarda's study of T_2-frames [1] and almost compact locales. Their description of almost compact locales is purely based on elements, but we present a different perspective in this talk. We shall adapt most of the results obtained by Yang in [2] and then describe almost compact locales using the notion of sublocales and open coverings, which is natural compared to how almost compact spaces are defined. We will also establish that the definition of almost compact locales is conservative in locales, meaning that a space is almost compact if and only if the locale of its open sets is almost compact. Lastly, we discuss some properties of almost compact locales that are not mentioned in the literature.

References:
1. Jan Paseka; Bohumil Šmarda, *T_2-frames and almost compact frames*, Czechoslovak Mathematical Journal, **42** (1992) 385—402.
2. Y. Yang, *Characterisations of quasi H-closed spaces*, Bull. Korean Math. Soc., **19** (1982) 35—37.

On the frame of z_r-ideals of $\mathcal{R}L$

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For a frame L, by $r(L)$ we mean the set of all regular elements (==non-zero divisors) of the ring $\mathcal{R}L$ of real-valued continuous functions on L. An ideal I of $\mathcal{R}L$ is said to be an r-ideal if for each $\varphi \in \mathcal{R}L$ and $\alpha \in r(L)$, $\varphi\alpha \in I$ implies that $\varphi \in I$ (see [1] and [2]). It is well known that an ideal I of $\mathcal{R}L$ is a z-ideal if whenever $\mathrm{coz}\lambda \leq \mathrm{coz}\varphi$ and $\varphi \in I$ implies $\lambda \in I$. An ideal which is both a z-ideal and an r-ideal is called a z_r-ideal (see [1] and [2]). The concept of z_r-ideals of $\mathcal{R}L$ is studied in the papers [1] and [2], wherein properties of these ideals are explored and used to characterize frames. Let $z_r\mathrm{Id}(\mathcal{R}L)$ be the lattice of z_r-ideals of $\mathcal{R}L$, ordered by inclusion. In this talk, we show that $z_r\mathrm{Id}(\mathcal{R}L)$ is a coherenttly normal frame.

References:
(1) M Abedi, M Matlabyana, and N Twala, *On some ideals of $\mathcal{R}L$*, Topology Proceedings (accepted), 2025.
(2) A.A Estaji, Z.N Khoshmardan, and R Pourkhandani, *On r-ideals of $\mathcal{R}(L)$*, Faculty of Sciences and Mathematics, University of Niˇs, Serbia (http://www.pmf.ni.ac.rs/filomat), **38** 11711–11729.

Quantum Groups, Knot Theory and Modular Categories

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 4,27,28,33

In his seminal 1989 paper (1), Edward Witten revealed a remarkable connection between topological quantum field theory and knot theory. His work inspired extensive research by both mathematicians and physicists, giving rise to what is now known as quantum topology. This talk provides a high-level overview of one aspect of this rich interplay: the equivalence between the category of finite-dimensional representations of the quantum group $U_q(\mathfrak{sl}_2)$ and the Temperley–Lieb–Jones category (2), constructed from tangle diagrams and the Kauffman bracket polynomial. We will outline the construction of both categories, discuss their structures as modular tensor categories (3), and construct a modular tensor equivalence between them.

References:
(1) E Witten, *Quantum Field Theory and the Jones polynomial*, Communications in Mathematical Physics, **121** (1989) 351–399.
(2) VG Turaev, *Quantum Invariants of Knots and 3-Manifolds (3rd Ed.)*, De Gruyter, 2016.
(3) P Etingof et. al., *Tensor Categories*, American Mathematical Society, 2015.

Calculating the Topological Entropy of Some Totally Disconnected Locally Compact Abelian Groups via the Associated Mastergraph

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SAMS Special Session: Categories, Algebra, Topology, and Logic
SAMS Classification: 15

The concept of topological entropy (see [2, 3]) arises in numerous areas of mathematics and applied science. In this talk, we investigate the topological entropy within the framework of topological abelian groups, with particular emphasis on the computation of the topological entropy of continuous automorphisms of totally disconnected locally compact abelian groups (see [3]). Furthermore, we introduce and analyze the notion of the mastergraph associated with these groups (see [1, 3]). We demonstrate that suitable structural constraints on the mastergraph entail the existence of continuous automorphisms possessing finite topological entropy. This relationship is illustrated through several examples involving well-known totally disconnected locally compact abelian groups.

References

[1] W. Herfort, K. H. Hofmann and F. G. Russo, *Periodic locally compact groups*, de Gruyter, 2019.

[2] F. G. Russo and O. Waka, *On locally compact groups of small topological entropy*, Acta Math. Hungar., Vol. 172, 2024, 62–79.

[3] O. Waka, *Small topological entropy of locally compact abelian groups restricting the mastergraph*, Topology and its Applications, Vol. 372, 2025, 109–230.

Small varieties of KR-algebras

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 19

The logic **KR** is a propositional relevance logic with a *Boolean*-style negation, witnessed in the theorem $(x \wedge \neg x) \to y$, yet it is unexpectedly undecidable (A. Urquhart, 1984). It also has some philosophically attractive features (Kerr, 2021). The variety KRA of all **KR**-*algebras* (which algebraizes **KR**) consists of Boolean algebras enriched with a special monoid operation $\circ$, called *fusion*. Despite this appealing structure, little is known about the lattice of subvarieties of KRA, particularly its smaller members. It follows from Moraschini *et al.* (2019) that this lattice has just two atoms, each generated as a variety by a finite 0-generated simple algebra. One of these generators is the two-element Boolean algebra **2** (wherein $\circ$ is $\wedge$), yielding the variety $\mathbb{V}(\mathbf{2})$ of Boolean algebras. The other, denoted by $\mathbf{D}_4$, has four elements.

We investigate the join-irreducible covers of these atoms in the subvariety lattice of KRA. In fact, $\mathbb{V}(\mathbf{2})$ has no such cover. Any such cover of $\mathbb{V}(\mathbf{D}_4)$ is generated by a simple 1-generated algebra that properly extends $\mathbf{D}_4$. We identify three covers of this kind, denoted here by $\mathbb{V}(\mathbf{K}_8)$, $\mathbb{V}(\mathbf{K}_8')$ and $\mathbb{V}(\mathbf{K}_{16})$. The algebras $\mathbf{K}_8$ and $\mathbf{K}_8'$ each have 8 elements, while $\mathbf{K}_{16}$ has 16.

We prove that $\mathbf{K}_8$ embeds into any simple **KR**-algebra $\mathbf{A}$, extending $\mathbf{D}_4$, that has a $\circ$-idempotent element lying outside $\mathbf{D}_4$. Also, if an algebra $\mathbf{A}$ of this kind has no such idempotent but has an element a such that $b \leq a \circ a = \neg a \circ \neg a$ for all elements b, then $\mathbf{K}_8'$ embeds into $\mathbf{A}$. We conjecture that $\mathbb{V}(\mathbf{K}_8)$, $\mathbb{V}(\mathbf{K}_8')$ and $\mathbb{V}(\mathbf{K}_{16})$ are the only join-irreducible covers of $\mathbb{V}(\mathbf{D}_4)$ in the subvariety lattice of KRA (or at least the only such tabular covers).

References:
(1) A. D. Kerr, *A plea for KR*, Synthese, **198** (2021) 3047–3071.
(2) T. Moraschini, J. G. Raftery, J. J. Wannenburg, *Varieties of De Morgan monoids: Minimality and irreducible algebras*, Journal of Pure and Applied Algebra, **223** (2019) 2780–2803.
(3) A. Urquhart, *The undecidability of entailment and relevant implication*, The Journal of Symbolic Logic, **49** (1984) 1059–1073.

Hollow Ideals and Hollow Elements

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SAMS Special Session: Categories, Algebra, Topology and Logic
SAMS Classification: 7

The study of strongly hollow ideals and their lattice-theoretic analogues provides a natural dual perspective to the classical notion of strong irreducibility. Beginning in the setting of commutative rings we will establish the basic properties of strongly hollow ideals under localization and ring homomorphisms. We give conditions related to strongly hollow ideals in which the union of prime ideals is itself a prime ideal. We extend the theory with the stronger notion of completely strongly hollow ideals and give conditions for the existence of such ideals. These stronger ideals will give a natural bijection to completely strongly irreducible ideals. We will study this bijection in quasi-local and Noetherian rings. We will end of the case in commutative rings by showing how the existence of a strongly hollow ideal is closely linked with the Jacobian radical of a ring. These concepts can be generalized to Multiplicative lattices. We investigate the interaction of strongly hollow elements with residuals, lattice localization, and quotients. We also discuss lattices which are generated by strongly hollow and completely strongly hollow elements and give a characterization of quasi-local multiplicative lattices by completely strongly hollow elements.

3 Special Session: Discrete Mathematics

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Average size of 1-nearly independent vertex sets

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SAMS Special Session: Discrete Mathematics
SAMS Classification: Combinatorics

A k-nearly independent vertex subset of a graph G is a set of vertices that induces a subgraph containing exactly k edges.

This talk will introduce the average size, $av_1(G)$ of the 1-nearly independent vertex subsets of graphs.

Characterisation of the n-vertex graphs with at least one edge that minimize or maximize av_1, as well as that of the trees of order n that achieve the minimum value of av_1, and asymptotically reach the maximum value will be discussed.

References:
(1) Dossou-Olory, Audace A. V., and Eric O. Andriantiana. *On the Average Size of 1-Nearly Independent Vertex Sets in Graphs.* arXiv, 2025, https://arxiv.org/abs/2510.23670
(2) E. O. Andriantiana and Z. B. Shozi. *The number of 1-nearly independent vertex subsets.* Quaestiones Mathematicae, **47** (2024) 2353—2373.
(3) E. O. D. Andriantiana, V. R. Misanantenaina, and S. Wagner. *The average size of independent sets.* of graphs. European Journal of Mathematics, **6** (2020) 561–576.

Extremal caterpillar-like trees relative to the Wiener index

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 6

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The Wiener index of a graph G is the sum of the distances between all vertex pairs in G. In this talk, we focus on *caterpillar-like trees* - trees with the property that all its branching vertices (vertices of degree greater than 2) lie on a path. We focus on the class of caterpillar-like trees with given order and other restrictions such as number of leaves and maximum degree. We determine the structures of those caterpillar-like trees that reach the minimum (or maximum) Wiener index under the prescribed restrictions.

References:

(1) H. Wiener, *Structural determination of paraffin boiling points*, Journal of the American Chemical Society **69** (1947) 17–20.
(2) A. Dobrymin, R. Entringer, and I. Gutman, *Wiener index of trees: Theory and applications*, Acta Applicandae Mathematica **66** (2001) 211–249.

Domination type parameters in 3-regular and 4-regular graphs

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 06

A set S of vertices in a graph G is a dominating set if every vertex in $V(G) \setminus S$ is adjacent to a vertex in S. An independent dominating set is a dominating set S with the additional property that the set S is independent. The domination number, $\gamma(G)$, of G is the minimum cardinality among all dominating set in G, while the independent domination number, $i(G)$, of G is the minimum cardinality among all independent dominating set in G. We discuss upper bounds on these parameters in 3-regular and 4-regular graphs. A thorough treatise on dominating sets can be found in (1).

References:
(1) T. W. Haynes, S. T. Hedetniemi, and M. A. Henning, *Domination in Graphs: Core Concepts* Series: Springer Monographs in Mathematics, Springer, Cham, 2023. xx + 644 pp.

Triangle-free graphs and the Ramsey number $R(3,t)$

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 6

The Ramsey number $R(3,t)$ asks for the smallest n guaranteeing that a triangle-free graph on n vertices has an independent set of size t. This is essentially the only non-trivial class of Ramsey numbers where the precise order of magnitude is known: it is know that $c\frac{t^2}{\log t} \le R(3,t) \le C\frac{t^2}{\log t}$ for some constants c and C. The upper bound was proven with $C = 1$ by Shearer in the 1980s, while a lower bound of the right order of magnitude was first proved in the 1990s by Kim, in a paper for which he received the Fulkerson prize. This constant has been improved in a number of increasingly long and technical papers until finally the constant $c = \frac{1}{3}$ was achieved recently by Campos, Jenssen, Michelen and Saharasbudhe. The true value of the constant has been conjectured to be $c = \frac{1}{2}$.

In this talk, we will describe a recent result of the presenter and collaborators on this problem. This is joint work with Zion Hefty (DU), Dylan King (Caltech) and Florian Pfender (UC Denver).

Rainbow colourings of certain paths

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SAMS Special Session: Discreet Mathematics
SAMS Classification: Combinatorics

Consider a connected simple graph $G = (V, E)$ with an edge colouring. For $u, v \in V(G)$, a $u - v$ path in G is a *rainbow path* if no colour is repeated on the edges of the path. Further, a *rainbow $u - v$ geodesic* is a rainbow path of shortest length between u and v. If G has every two vertices connected by a rainbow path, then G is said to be a *rainbow connected graph*. Similarly, if G has every two vertices connected by a rainbow geodesic, then G is said to be a *strong rainbow connected graph*. The *rainbow connection number*, denoted (G), is the minimum number of colours needed to make G rainbow connected. The *strong rainbow connection number*, denoted (G), is the minimum number of colours needed to make G strong rainbow connected. We present work showing colourings of several families of graphs.

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Mokoena's Generalised Lambert Transform

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 22, 16

The late Joseph Albert Mashite Mokoena was the first Black South African to complete a PhD in mathematics which he received in 1958 from the University of the Witwatersrand under the supervision of James Hyslop for a thesis titled "An inversion of a General Class of Lambert-type Integral Transform" [1]. In this talk, I will discuss the work of Mokoena on a generalisation of the Lambert transform. His work is based on that of Widder [2], who introduced a generalisation of the Laplace transform, later known as the Lambert transform. In the literature, the Lambert transform has various forms of generalisation. A widely known and cited work on this, was done by Goldberg [3], Widder's PhD student. We note the similarities and the differences of both generalisations given by Goldberg and Mokoena. A recent paper by González and Negrín [4] introduced an approach to the Riemann hypothesis using a form of Lambert transform as given by Goldberg. Given this recent important interest, the work of Mokoena should be recognised by the mathematical community, and this is our first attempt to do so.

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Variations on Sheldon primes

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 22

A Sheldon prime is a prime p_n such that the product of the digits of p_n (in base 10) is the index n of p_n in the sequence of prime numbers and such that furthermore the reversal of p_n (also in base 10) is the prime p_m, where m is the reversal of n in base 10. It was recently proved by Pomerance and Spicer that 73 is the only Sheldon prime. We generalise this notion by allowing possibly different bases b, b' such that the product of the digits of p_n in base b is n, the reversal of p_n in base b is p_m and n, m are reversals of each other in base b'. We present an heuristic that there should be infinitely many quadruples (b, b', p_n, p_m) with the above properties and present some examples.

The Annihilating-ideal Graph of a Commutative Ring with or without S-vertices

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SAMS Special Session: Discrete mathematics
SAMS Classification: Combinatorics

Let R be a commutative ring with $1 \neq 0$ and $\mathbb{A}(R)$ the set of annihilator ideals of R. An ideal I of R is an annihilator ideal if there exists a nonzero ideal J in R such that $IJ = (0)$. The annihilating-ideal graph of R, denoted by $\mathbb{AG}(R)$, is defined to be the graph with the vertex set $\mathbb{A}(R)^* = \mathbb{A}(R) \setminus \{(0)\}$ and two distinct vertices I and J are adjacent if and only if $IJ = (0)$. A vertex a in a simple graph G is said to be a Smarandache vertex (or S-vertex for short) provided that there exist three distinct vertices x, y, and b (all different from a) in G such that x—a, a—b, and b—y are edges in G, but there is no edge between x and y. In this interdisciplinary subject, we investigate the interplay between the algebraic and graph-theoretic properties of $\mathbb{AG}(R)$ using the notion of the Smarandache vertices in connection with the existence or nonexistence of S-vertices in the graph $\mathbb{AG}(R)$. We will show that a conjecture related to the weakly perfectness of $\mathbb{AG}(R)$ is true when the graph has no S-vertices. It is shown that how the existence of an S-vertex in $\Gamma(R)$, the zero-divisor graph of R, implies the existence of an S-vertex in $\mathbb{AG}(R)$. We characterize rings R when $\mathrm{gr}(\mathbb{AG}(R)) \geq 4$, and so we characterize rings whose annihilating-ideal graphs are bipartite. There is also a discussion on a relationship between the diameter, girth, and S-vertices of $\Gamma(R)$ and $\mathbb{AG}(R)$ (in particular, when R is a polynomial or a power series ring).

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On the Computation of the Projective Character Tables of the Maximal Subgroups Related to the Sporadic Simple Groups $\mathbb{H}S$ and Its Automorphism Group $\mathbb{H}S : 2$

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The computation of character tables is very important in understanding the structure of finite group, particularly in the case of maximal subgroups of the sporadic simple groups. While ordinary character tables for many of these subgroups are documented, the projective character tables remain incomplete. The main aim of this project is to construct the full cover group of each of the maximal subgroups of the simple sporadic groups HS and their automorphism groups $HS : 2$. From the full cover group, we can then obtain all the irreducible projective characters with associate factor sets of a maximal subgroup. A GAP routine based on the theory of lifting projective characters of a finite group G to its so-called Representation group (full cover group of G) will mostly be used. In addition, we will also apply the Fischer-Clifford matrices method to construct the full cover groups (central extensions) of some of the mentioned maximal subgroups.In this study most of the computations will be done by using GAP and, where appropriate, MAGMA. This study will not only fills important gaps in the Atlas of Group Representations but also advances computational group theory through the expansion of known character data for complex finite groups.

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A Note on Fibonacci Numbers and the Golden Ratio of Order k

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SAMS Special Session: Discrete mathematics
SAMS Classification: 22

We define and study the notion of the golden ratio of order $k \geq 0$, denoted ϕ_k, as a generalized form of the golden ratio ϕ for any real number $k \geq 0$. We show that similar to the special case of ϕ and its conjugate ψ, ϕ_k and ψ_k are the two distinct roots of a quadratic polynomial for any fixed real $k \geq 0$. We express some numerical and algebraic properties of ϕ_k and ψ_k and write their relations to ϕ and ψ, respectively, with some examples for some special values of k. In particular, it is shown that $\phi_k = \phi$ and $\psi_k = \psi$ if and only if $k = 0$. We show that $\mathbb{Z}[(k+1)\phi_k]$ is a subring of the ring $\mathbb{Z}[\phi]$ for any nonnegative integer k. We will define the golden rectangle of order k (or k-golden rectangle for short) with a class of examples for all $k \geq 0$. We also discuss some cases of two Fibonacci numbers in connection to the golden ratio. We will show that the ratio of height to width of the pages of the Gutenberg Bible is the golden ratio of order $k \neq 0$. Actually, some erroneous ideas and examples of disputed observations related to the golden ratio are good reasons to apply ϕ_k to improve the measurements regarding ϕ for some $k \neq 0$. Finally, we end the paper by posing a question related to the Penrose tiling and quasicrystals in connection to the golden ratio of order $k > 0$.

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Constructing Invariant t-Designs from Twisted and Congruence Maximal Subgroups of $PSp_4(q)$

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 6

In this presentation, we construct combinatorial designs arising from the twisted and congruence maximal subgroups of finite primitive groups, using their conjugacy classes and character tables. Our approach follows the framework established by Key and Moori, widely known as the second Key-Moori method. We examine subgroup structure, cosets, orbit decompositions, fixed points, and character data to determine permutation characters associated with primitive group actions. While our primary focus is on the projective symplectic group $PSp_4(q)$, where q is a prime power, the techniques are general and applicable to other primitive groups as well.

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A Unified generalization of Stirling numbers

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 06

In this talk we examine a new kind of unified generalization of Stirling numbers. We show how it is a generalization of several extensions of stirling numbers. We prove several of their combinatorial identities. We also discuss its possible applications to other areas of combinatorics as well.

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Designs invariant under the unitary group $U(3,q)$

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SAMS Special Session: Discrete Mathematics
SAMS Classification:15

In this presentation, we outline a method for constructing designs invariant under primitive permutation simple groups. We will use Key-Moori Method 2 to construct some designs from maximal subgroups and conjugacy classes of unitary groups. The group will act as an automorphism group on the designs, premitive on blocks and transitive on points In the paper [3] the authors used the same approach to find designs of the Suzuki Groups.
We first study the general structure of unitary groups using MAGMA[1] and Atlas[2] for basic structures on smaller groups, we then generalise to bigger unitary groups. In addition we use the obtained designs to construct linear codes.

References:

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Trees with extremal number of split and non-split dominating sets

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SAMS Classification: 06

A set D of the vertices of a graph G is called a *dominating set* of G if every vertex of G is either in D or has a neighbour in D. Bród and Skupień studied the number of dominating sets, $\delta(T)$, T being a tree. They showed that the star is the unique tree with maximum $\delta(T)$ on n vertices (except when $n = 4$ and $n = 5$, where the path also has the same $\delta(T)$). Interestingly, a rich class of trees attain the minimum $\delta(T)$.

A dominating set D of G is a *split dominating set* (SDS) of G, if the vertex set of G without D is a disconnected graph, otherwise it is a *non-split dominating set* (NSDS) of G.

Given a tree T with n vertices, we determine the number of SDSs, $\delta_s(T)$, and the number of NSDSs, $\delta_{ns}(T)$ of T. The maximum $\delta_s(T)$ as well as the minimum $\delta_{ns}(T)$ is always attained by the star. Interestingly, the minimum $\delta_s(T)$ is attained by at most two of the trees that attain the minimum $\delta(T)$. The maximum $\delta_{ns}(T)$ is attained by either a *subdivided star* or a *double subdivided star*.

Symmetry Breaking in Trees

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 6

A graph G is *D-distinguishable* if there is a labeling of its vertices with D labels such that only the trivial automorphism of G preserves the labels. The *distinguishing number* of G is the minimum value D for which G is D-distinguishable. The *fixing number* of G is the minimum cardinality of a subset of the vertices of G which is fixed pointwise only by the trivial automorphism. In this talk, we consider both symmetry breaking parameters, distinguishing and fixing, in tree graphs.

The k-dominating graph of triangle free graphs

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SAMS Classification: 6

A dominating set S of a graph is a set of vertices, such that every vertex not in S is adjacent to a vertex in S. The domination number and upper domination number of a graph G is denoted by $\gamma(G)$ and $\Gamma(G)$, respectively. The k-dominating graph of G, $D_k(G)$, is the graph whose vertices corresponds to all the dominating sets of G of cardinality at most $k \geq \gamma(G)$ and where two vertices are adjacent if and only if the corresponding dominating sets of G differ either by adding or deleting a single vertex. We are interested in when the graph $D_k(G)$ is connected. In order to study the connected ness, let the smallest integer ϵ for which $D_k(G)$ is connected for all $k \geq \epsilon$ be denoted by $d_0(G)$.

It is known that $\Gamma(G) + 1 \leq d_0(G) \leq \min\{|V(G)|, \Gamma(G) + \gamma(G)\}$ and it was shown by Mynhardt et. al. that there exist an infinite class of graphs that attains this upper bound. However, these graphs and others for which $d_0(G) > \Gamma(G) + 1$ all contain triangles. So what happens when the graph is triangle free? It has been shown that $d_0(G) = \Gamma(G) + 1$ for all bipartite graphs and it was conjectured that this stays true for all triangle free graphs. In this talk we present triangle free graphs which exceeds this lower bound.

References:

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An Impartial Game on Conjugacy Classes of Finite Groups

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Joint work with Hadiseh Saydi

SAMS Special Session: Discrete Mathematics
SAMS Classification: 06, 15

We study a variant of the impartial avoidance game Do Not Generate (DNG), played on a single conjugacy class of a finite group. In the original DNG game, players alternately select elements from a finite group, and the player who first completes a generating set loses. Our variant restricts the moves to a conjugacy class that generates the group, ensuring the game always terminates. This setting naturally includes finite simple groups. We analyse strategies for this restricted game, present winning strategies for certain families of groups, and explore the associated Sprague–Grundy numbers.

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Some Designs Under The Dicyclic Group Algebra

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 15

We introduce a reformulation of the Key-Moori Method 2 for construction codes and designs by finite groups. Our main example for this method is about the dicyclic group. We construct designs using the dicyclic group. For this aim, it is necessary to check some details about the subgroups of the dicyclic group.

References:
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Compression with wildcards all k-models of a Binary Decision Diagram

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 06

Given the DNF form of a Boolean function ϕ on n variables, e.g.,

$$\phi = \overline{x}_1 x_2 \overline{x}_3 \vee \overline{x}_1 \overline{x}_2 x_3 \vee \overline{x}_1 x_3 \vee x_2 x_3 x_4 \vee x_4 x_5 x_6 \vee \overline{x}_4 \overline{x}_5 \overline{x}_6 \vee x_1 x_3 \overline{x}_4 \vee x_3 x_5 \overline{x}_6,$$

deciding whether the equation $\phi = 1$ is consistent or not is known to be NP [1]. However, given a Binary Decision Diagram [2] of the Boolean function ϕ, all N many ϕ-models of Hamming-weight k can be enumerated in polynomial time [3]. In this project, based on novel wildcards introduced in [4], we obtain a compressed enumeration of these models. Further, we randomly generate Boolean functions in DNF form on n variables with exactly p clauses of length q and calculate N.

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On the generator graph of a cyclic group

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 6

In this talk, we continue the study of the generator graph of a group. In 2023, Tacbobo [1] defined the generator graph of a nontrivial group to be the graph whose vertices are the elements of the group, with two vertices being adjacent if at least one of them is a generator of the group. We present explicit formulas for some topological indices of the generator graph of a cyclic group with $n \geq 2$ elements and whose set of generators is S, expressed in terms of n and $|S|$. Furthermore, we determine the metric dimension of the generator graph of a nontrivial cyclic group as a function of its order n.

References:
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Cases of the exponential local-global principle

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SAMS Special Session: Discrete Mathematics
SAMS Classification: 22

The Skolem conjecture, or exponential local-global principle, is the following statement (Bilu et al. 2022):

For some positive integer s, suppose that some two-way infinite sequence $t = (\ldots, t(-2), t(-1), t(0), t(1), t(2), \ldots)$ consists of numbers in $\mathbb{Z}[1/s]$ and satisfies a linear recurrence for which the characteristic polynomial has only simple nonzero roots. Suppose that for all positive integers m that are coprime to s, some term of t is a multiple of m in $\mathbb{Z}[1/s]$. It follows that some term of t is zero.

The general conjecture is open. In this talk, results are presented concerning some relatively simple examples of the exponential local-global principle.

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4 Special Session: Financial Mathematics

Session organisers: Farai Mhlanga (mhlanga@ul.ac.za)

Stochastic Differential Reinsurance Game Formulation and Investment Problem Under Various Utility Functions

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SAMS Special session: Financial Mathematics
SAMS Classification: 25

This talk presents a stochastic differential game between two competing insurance companies that simultaneously engage in reinsurance and investment in risky financial assets. Each insurer maximizes the expected utility of terminal wealth under exponential, logarithmic, and power utility functions, reflecting heterogeneous risk preferences. The surplus processes are modeled by correlated Brownian motions, thereby capturing interdependence in both underwriting risk and financial markets. The Hamilton-Jacobi-Bellman (HJB) and Fleming-Bellman-Isaacs (FBI) equations are used to characterize the strategic interaction, and Nash equilibrium strategies are derived, ensuring that no insurer can unilaterally improve its position.

The results highlight the critical influence of correlation between surplus processes: stronger correlation amplifies competition and limits diversification, while weaker correlation allows more aggressive investment strategies. Simulation results further show that insurers with greater initial reserves retain more risk and invest heavily in risky assets, while weaker competitors rely on reinsurance for protection. The study extends existing literature by jointly considering risky investment, surplus correlation, and heterogeneous risk preferences, offering theoretical contributions and practical insights for

insurers operating in competitive and uncertain markets.

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Robust Portfolio Optimization Under Extreme Market Strain

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SAMS Special Session: Financial Mathematics
SAMS Classification: 12

The Classical Markowitz Mean-Variance model relies on the assumptions of normally distributed returns and constant correlations, performing well in stable markets but failing during periods of extreme stress. To address this limitation, we propose an extension of the model that enhances its resilience to market instability. Our framework integrates three key techniques: Conditional Value-at-Risk (CVaR) to manage tail-risk, Dynamic Conditional Correlation (DCC) to capture time-varying volatility and asset correlations, and robust optimization to mitigate sensitivity to estimation errors. The performance of this extended model will be empirically evaluated against the traditional Markowitz approach and standard benchmarks using historical data.

Regular-impulse control for air pollution management: A stochastic differential game approach

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SAMS Special Session: Financial Mathematics
SAMS Classification: 25

The paper investigates a finite-horizon, two-player nonzero-sum stochastic differential game describing a pollution abatement initiative. A mining company (Player 1) controls a piecewise continuous resource extraction rate, while the government (Player 2) applies impulse control penalty interventions to compel the company to reduce pollution levels. Both players aim to minimize the expected total discounted operational costs and intervention costs incurred over the finite horizon. The main results are formulated as a stochastic impulse control problem. This problem is approached via quasi-variational inequalities (QVIs), which provide an analytical characterization of the Nash equilibrium strategies. To illustrate the theoretical results, a scalar linear–quadratic example is discussed, supported by numerical experiments.

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Continuous-Time Portfolio Rebalancing Under Transaction Costs: A Stochastic Control Formulation

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SAMS Special Session: Financial Mathematics
SAMS Classification:

This study develops an initial stochastic control framework for portfolio rebalancing in the presence of transaction costs [1]. The aim is to construct a simplified continuous-time model that captures the fundamental trade-off between maintaining desired portfolio allocations and limiting the costs incurred when adjusting positions. The analysis centres on the formulation of the value function and the derivation of the corresponding Hamilton–Jacobi–Bellman (HJB) equation that governs the optimal rebalancing strategy [2]. A Finite method is applied to approximate the solution of this equation and to examine how different transaction cost parameters influence the resulting control structure [3]. The preliminary findings offer insight into the qualitative behaviour of optimal rebalancing decisions and provide a basis for the broader development of a comprehensive approach to portfolio optimisation under transaction costs.

References:
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(2) Dai, Min, and Yifei Zhong. *Penalty methods for continuous-time portfolio selection with proportional transaction costs*, Available at SSRN 1210105 (2008).
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A stochastic control model of investment, production and consumption with transaction costs

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SAMS Special Session: Financial Mathematics
SAMS Classification:

We examine an infinite-horizon stochastic control problem for maximising the expected discounted utility of consumption through optimal investment and consumption. The economic environment is characterised by random fluctuations, with the unit price of capital is governed by a geometric Brownian motion and the income from production is also subject to random Brownian fluctuations. While the frictionless benchmark for this problem was solved by Fleming and Pang (1) and Han and Yi (2), our analysis extends the framework by incorporating proportional transaction costs, defined as a fixed percentage charge on all transactions.

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An optimal investment and consumption model with transaction costs and regime-switching: A stochastic differential utility approach

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SAMS Special Session: Financial Mathematics

SAMS Classification: 20

This paper investigates the optimal investment and consumption problem in financial markets characterized by regime-switching dynamics and proportional transaction costs, both of which introduce realistic frictions absent in classical models. Unlike Merton's frictionless framework, our model captures market features such as volatility shifts and trading costs by allowing asset returns and volatilities to depend on a finite-state continuous-time Markov chain. The investor's preferences are modelled when the utility is of the stochastic differential type such as Epstein-Zin stochastic differential utility, enabling a separation between risk aversion and intertemporal substitution. We formulate the resulting control problem, define admissible strategies, and derive the corresponding Hamilton-Jacobi-Bellman (HJB) equation for the value function under this general setting. A candidate solution is proposed, and the structure of the HJB equation highlights the complex interplay between consumption, investment, regime-switching, and transaction costs. The model provides a more realistic foundation for developing optimal strategies aligned with observed investor behaviour.

References:
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Mean Variance Hedging in Incomplete Financial Markets

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SAMS Special Session: Financial Mathematics
SAMS Classification: 23,25,26

In this talk, we investigate the mean variance hedging in incomplete markets, constructing self-financing trading strategies that minimise the expected squared hedging error. We derive explicit, implementable strategies using the Galtchouk-Kunita-Watanabe projection, the variance-optimal martingale measure and a change of numéraire. The approach is applied to basis risk hedging, where closed-form solutions are obtained and validated through comprehensive simulations.

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An efficient neural network approach for option valuation under fractional dynamics

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SAMS Special Session: Financial Mathematics
SAMS Classification: 12

We present a novel numerical method for solving time-fractional Black-Scholes equations for pricing financial derivatives. Our method makes use of machine learning approaches, in particular, Physics Informed Neural Networks (PINNs). Due to the inherent non-local and non-smooth characteristics of fractional derivatives, the conventional numerical methods often struggle to provide reliable solutions of fractional partial differential equations. This challenge can be partly resolved by using PINNs on refined time-domain partitions. These refinements enable the network to simultaneously learn both the underlying financial dynamics and the fractional derivative operators. In this talk, we will demonstrate through various numerical simulation results that our PINN-based method provides solution which is very robust and cost-effective when handling high-frequency or noisy data.

Wasserstein distributional risk of convex payoffs

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SAMS Special Session: Financial Mathematics
SAMS Classification: 12, 11

The dual formula for a distributionally robust expected value under Wasserstein distance requires the computation of the λc-transform of the payoff function f. While not always an intuitive object, in specific cases simple representations are available. In particular, if c is the quadratic cost function on $\mathbb{R}^n$ and f a pointwise maximum of affine functions, the link between the λc-transform and the Legendre transform allows one to derive an intuitive representation. We apply this to distributionally robust – with respect to ambiguity tolerance parameter θ and perturbations of the risk-neutral distribution – calculations of a three-asset portfolio containing call and put options.

5 Special Session: Functional Analysis and Operator Theory

Session organisers: Christian Budde (BuddeCJ@ufs.ac.za)

C_0-semigroups with a parameter and their applications in kinetic models with spatial transport

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 3,11

Many physical processes are driven by several mechanisms, each acting on separate groups of variables, with the remaining ones being parameters. Examples are offered by spatially inhomogeneous kinetic processes, where the latter, driven by an internal variable, such as mass or energy of interacting particles, occur against the backdrop of a moving medium. Thus, the evolution of the system is governed by the sum of two operators, one acting on the internal variable with the spatial variable occurring as a parameter, and the other conversely. We provide a simple criterion ensuring the existence of a C_0-semigroup describing the full process, and provide examples of fragmentation-coagulation equations with spatial diffusion or in a moving medium.

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Commutativity of operator algebras

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification:

We call an operator algebra A *reversible* if A with reversed multiplication is also an abstract operator algebra (in the modern operator space sense). This class of operator algebras is intimately related to the symmetric operator algebras: the subalgebras of $B(H)$ on which the transpose map is a complete isometry. In previous work we studied the unital case, where reversibility is equivalent to commutativity. Not every reversible or symmetric operator algebra is commutative, however we show that they all are 3-commutative. That is, order does not matter in the product of three or more elements from A. The canonical anticommutation relations from mathematical physics play a significant role.

Asymmetry, inner products and partial orders

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

Inner products generate norms, but not all norms are induced by inner products. It is possible to modify the definition of an inner product, in the process removing the symmetry condition, to obtain what is usually known as a semi-inner product and to show that every norm can be generated by such a semi-inner product. It could therefore be argued that semi-inner products should rather be called asymmetric inner products.

In this talk we look at other possible candidates for this name, more general in the sense that they generate asymmetric norms, rather than norms. Associated with the asymmetric norms are cones, which in their turn induce partial orders on the underlying vector space. We investigate the relationship between these orders and the asymmetric inner products, and in particular the cases in which we obtain lattice-ordered inner product spaces.

An extension of Schauder's fixed point theorem with application to solvability of a system of differential equations

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

In this work, we consider a new class of non-self mappings, called proximal condensing operators and study the existence of best proximity points for such class of maps in reflexive and strictly convex Banach spaces by applying a suitable measure of noncompactness. In this way, a real generalization of a well-known Schauder's fixed point theorem will be concluded. As an application of our existence theorem, by renorming the Banach space $\mathcal{C}[0, 1]$ consists of all real valued and continuous functions defined on $[0, 1]$ to obtain a strictly convex Banach space, we survey the existence of an optimal solution for a system of differential equations.

References:
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Representation Theorems of Truncated Vector Lattices

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 34

Truncated vector lattices of functions have been introduced by Marshall Stone as the natural framework for pre-integrals to be represented as integrals. Later, David Fremlin obtained a representation of certain abstract normed vector lattices as truncated vector lattices of bounded functions ("truncated vector lattices" is due to Fremlin). About ten years ago, Richard Ball gave a remarkable intrinsic axiomatization of this concept. Over the past seven years, the Tunisian school made a significant contribution on this old-new theory. In this talk, I intend to give some representation theorems of truncated vector lattices from particular cases to the general one.

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A Strongly convergent algorithm for approximating a common solution to fixed point and mixed equilibrium problems

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In this paper, we propose an inertial iterative method for solving a common solution to the fixed point and mixed equilibrium problem in Hilbert spaces. We prove the sequence generated by the proposed algorithm strongly converges to an element in the solution set of mixed equilibrium problems of a pair of bi-function, which is also the solution to a fixed point of demicontractive mapping. Finally, we give some numerical experiments to support our main result. Our result extends and generalizes some earlier announced results in the literature.

SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

A von Neumann algebraic approach to Quantum Theory on curved spacetime

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 10,11,20

We present the AQFT (Algebraic Quantum Field Theory) framework in terms of von Neumann algebras. In particular, this approach allows for a locally covariant categorical description of AQFT which moreover satisfies the additivity property and provides a natural and intrinsic framework for a description of entanglement. Turning to dynamical aspects of QFT we show that Killing local flows may be lifted to the algebraic setting in curved spacetime. Furthermore, conditions under which quantum Lie derivatives of such local flows exist are provided. The central question that then emerges is how such quantum local flows might be described in interesting representations. We show that quasi-free representations of Weyl algebras fit the presented framework perfectly.

Wold decomposition and structure for isometries with equal range

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SAMS Special Session:. Functional Analysis and Operator Theory
SAMS Classification: 11

Let $n \geq 2$, and let $V = (V_1, \ldots, V_n)$ be an n-tuple of isometries acting on a Hilbert space $\mathcal{H}$. We call V an n-tuple of *isometries with equal range* on $\mathcal{H}$ if the conditions

$$V_i^{m_i} V_j^{m_j} \mathcal{H} = V_j^{m_j} V_i^{m_i} \mathcal{H} \quad \text{and} \quad V_i^{*m_i} V_j^{m_j} \mathcal{H} = V_j^{m_j} V_i^{*m_i} \mathcal{H}$$

hold for $m_i, m_j \in \mathbb{Z}_+$, where $1 \leq i < j \leq n$.

In this talk, we prove that every n-tuple of isometries with equal range admits a unique Wold decomposition. Inspiring from the analytic representation for a single isometry, we derive the analytic model for an n-tuple of isometries with equal range. As a consequence, we show that the wandering data are complete unitary invariants for the above class. Our findings consolidate and generalize all previous results on the decomposition for tuples of isometries found in the existing literature. This is a joint work with Amit Maji.

References:
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Relationship between strictly convex and reflexivity properties in a Banach Space using BP cone

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SAMS Special session: Functional Analysis and Operator Theory
SAMS Calssification number 11

In this talk, we discuss the equivalence between strictly convex and reflexivity properties in a Banach space as conjectured by V.L. Klee. There has been some research on characterization of these properties in terms of a BP cone. For instance, a strictly convex is characterized by the absence of a BP cone C_f^1 which contains atleast two linearly independent vectors while a reflexive space by the existence of a non-trivial BP cone C_f^1. A BP cone is a cone defined on a normed space as $C_f = \{x \in X : \|x\| \leq f(x)\}$, where $f \in X^*$ (the dual of X). For $f \in X^*$ such that $\|f\|^* = 1$, we have

$$C_f^1 = \{x \in X : \|x\| = f(x)\}.$$

A BP cone is characterized by a cone with a bounded base. We establish the relationship between strictly convex and reflexivity using BP cone C_f^1.

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Locally band preserving functions in Dedekind complete Φ-algebras

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Roelands and Schwanke recently developed a theory of differentiation on Φ-algebras in (1). We have shown that a super order differentiable function f defined on an order interval has the following property: whenever $\mathbb{P}(x) = \mathbb{P}(y)$ for $x, y \in \mathrm{dom}(f)$ and a band projection $\mathbb{P}$, we have that $\mathbb{P}(f(x)) = \mathbb{P}(f(y))$. Functions with this property are called *locally band preserving*, and we study this property to continue the work started in (1). We discuss how many classical results, such as the extreme value theorem and mean value theorem, can be recovered for locally band preserving functions in Dedekind complete Φ-algebras.

References:
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Novel iterative method with golden ratio for solving quasimonotone variational inequalities

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We propose a unified iterative method that integrates the subgradient extragradient method with the golden ratio technique, tailored for solving quasi-monotone variational inequalities in real Hilbert spaces. The proposed method incorporates a dynamic stepsize rule equipped with both linesearch and self-adaptive techniques, eliminating the need for prior knowledge of the Lipschitz constants. We establish both weak and linear convergence of the proposed method under some mild conditions, extending known results from monotone and pseudo-monotone to quasi-monotone setting. Application to image restoration problem and some numerical examples to demonstrate the efficiency and robustness of our algorithm were discussed. Our findings improve, unify, generalize and extend the results in [1, 2, 3, 4, 5].

References

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Inertial self-adaptive algorithm for solving non-Lipschitz Monotone variational inclusion problems

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In this paper, we introduce a modified Tseng extragradient for solving monotone variational inclusion problem in real Hilbert spaces. Our method does not require the associated single-valued operator to be Lipschitz continuous. Rather, it requires uniform continuity which is a weaker assumption. We prove the strong convergence of our new method under some condition on the control parameter. We carry out numerical experiment to show the computational advantage of the new method over some existing methods in the literature. Our results extends and generalizes some well known results in literature.

SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

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A characterization of maps between operator systems which preserve boundary representations

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Let S be an operator system, contained in the C*-algebra it generates, $C^*(S)$. A *boundary representation* of S, is an irreducible representation of $C^*(S)$ on a Hilbert space, whose restriction to S has a unique completely positive extension to $C^*(S)$. Every operator system has sufficiently many boundary representations to the generate the C*-envelope of S [1], which acts as a non-commutative analogue of the Shilov boundary of a uniform algebra. Using a characterization of linear maps that preserve pure states between C*-algebras given in [2], we give a characterization of unital completely positive maps between operators systems which preserve boundary representations.

References:
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Ornstein-Uhlenbeck semigroup and application to stochastic differential equations

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 10

We will investigate the so-called Ornstein-Uhlenbeck semigroup which is linked to the Ornstein-Uhlenbeck operator. Those objects are of interest in several fields, from quantum mechanics to stochastic analysis, control theory and partial differential equations. Evolution equations driven by Ornstein–Uhlenbeck operators are the Kolmogorov equations of linear stochastic ODEs, and they are one of the few examples of multidimensional linear parabolic equations for which a resolvent kernel is explicitly known. We will also sketch the derivation of the explicit representation of the semigroup generated by the Ornstein-Uhlenbeck operator.

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Perturbation results in the Fredholm theory of Banach algebras

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification Number: 11

Perturbation results pop up regularly in the study of invertibility in Banach algebras. A classical example of this is the proof that the set of invertible elements is open, which uses the following perturbation result: If $||a|| \leq 1$, then $1_A - a$ is invertible with $(1_A - a)^{-1} = \sum_{k=0}^{\infty} a^k$. Another version of this result was formulated in rings by Roch and Silbermann:
If x is nilpotent and commutes with a, then a is invertible if and only if $a + x$ is invertible.

In this talk, we will explore various perturbation results analogous to this observation in the context of Fredholm theory. Given that Fredholm theory studies various concepts relative to Banach algebra homomorphisms that are related to invertibility, this will prove to be a prime area to obtain such results.

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Banach algebra mappings preserving the invertibility of differences

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11, 30, 33

Let A and B be complex unital Banach algebras, and let $\varphi, \psi : A \to B$ be surjective mappings, with $\varphi(0) = \psi(0) = 0$, which together preserve the invertibility of differences in both directions; that is, for any $x, y \in A$, $x - y$ is invertible in A if and only if $\varphi(x) - \psi(y)$ is invertible in B. If A is semisimple, we can prove that both φ and ψ preserve adjacency (differences of rank one) in both directions. By exploiting this connection to adjacency preserving mappings, we shall present in this talk a complete classification of the form of φ and ψ whenever A belongs to a large class of Banach algebras (one which includes $\mathcal{L}(X)$ with X infinite-dimensional). More specifically, we show in this case that $\varphi(x) = \psi(x) = uJ(x)$ for each $x \in A$, where u is a fixed invertible element in B and $J : A \to B$ is a linear or conjugate-linear Jordan-isomorphism.

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Square closed vector lattices

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

We explore multiplication from an order-theoretical perspective in this talk. This journey naturally leads to the notion of square closed vector lattices, which are vector lattices that are closed under a specific infinite supremum. Particularly, we discuss how an Archimedean vector lattice is a Φ-algebra (loosely speaking, a vector lattice with a compatible multiplication) if and only if it is square closed. We then illustrate by example how this result is an efficient tool for testing whether or not an Archimedean vector lattice is a Φ-algebra, and we discover some new Φ-algebras along the way.

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Some fixed point theorems of α-partially nonexpansive mappings

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

This work presents a study of the class of α-partially nonexpansive mappings. We examine the relationship between this class and other significant generalizations of nonexpansive mappings found in the literature, notably those defined by Suzuki, Llorens-Fuster, and Dhompongsa et al.

We prove several existence and convergence theorems for these mappings in Banach spaces, with a particular focus on spaces possessing normal structure. A key result demonstrates the existence of a common fixed point for a family of commuting α-partially nonexpansive self-mappings.

Our findings directly extend and generalize several well-known results, including the central theorems of Suzuki (2008), Llorens-Fuster (2022), and Dhompongsa et al. (2009). The presentation is supported by illustrative examples that validate our definitions and main conclusions. Finally, we propose an open problem regarding the existence of fixed points for α-partially nonexpansive mappings in the setting of uniformly nonsquare Banach spaces.

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Fixed Point Theorems in Cone θ-type multiplicative metric spaces

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

Fixed point theory forms a cornerstone of nonlinear analysis with extensive applications in various scientific disciplines. Motivated by the generalizations of classical metric spaces, this paper introduces and investigates the concept of cone θ-type multiplicative metric spaces and their extensions to 2- and 3-variable frameworks. Building upon the foundations of θ-metric spaces and multiplicative metric spaces, we define solid multiplicative cones in ordered Banach algebras and establish corresponding topological and algebraic properties. Using these structures, we derive several fixed point theorems for self-mappings satisfying cone-valued θ-type multiplicative contractive conditions. The results generalize well-known contraction principles of Banach, Kannan, and Chatterjea in the context of multiplicative and cone-valued metrics. Illustrative examples are provided to validate the introduced concepts and demonstrate their applicability in broader analytical settings.

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Non-commutative mixed normed spaces

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

Mixed L^p norm spaces were first introduced by Benedek and Panzone in (2). In general, let E and F be Banach function spaces on the measure spaces (X, Σ, μ) and (Y, Λ, ν) respectively. We define the mixed norm space $E[F]$ to be the space of $\mu \times \nu$ measurable functions such that $\| \, \|f(s,t)\|_{E,s}\|_{F,s} < \infty$ (see (3) and (4)).

Let $\mathcal{M}$ and $\mathcal{N}$ be semi-finite von Neumann algebras equipped with faithful, normal, semi-finite traces. We would like to make sense of non-commutative generalisations of mixed norm spaces with respect to $\mathcal{N}$ and $\mathcal{M}$. We discuss previous attempts at defining such spaces, discuss various strategies to tackle the general case, and show that one can obtain a working theory in the case when one of the von Neumann algebras is abelian.

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Spectral properties of unbounded Toeplitz operators with rational matrix symbols

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

Using state space methods, in this talk we derive spectral properties of operators from a class of unbounded Toeplitz operators with rational matrix symbols. Along the way, we derive a Wiener-Hopf type factorization using algebraic Riccati equations, with stable and semi-stable solutions, to characterize invertibility. The talk is based on joint work with Gilbert Groenewald, Jacob Jaftha and André Ran collected in [1,2].

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Algebraic and Analytic Free Objects

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SAMS Classification: 11

Free objects play a central role in algebra. For a given class of mathematical structures (e.g. rings, Banach spaces, locally convex spaces), the free objects are the *generic* representatives of the given class since free objects only possess the structure imposed by the *'axioms'* of the class. From universal algebra, any variety (equational class) is generated by a free object over a countably infinite set of generators.

By contrast, free objects have not been as influential in analysis. In the field of Positivity, both algebraic and analytic free objects have received attention, including free vector lattice algebras and free Banach lattice algebras. However, the abstract theory of universal algebra needs to be invoked to obtain the *existence* of these free objects, as direct constructions remain elusive. The invocation of universal algebra is often viewed as a *crutch* for the fact that such direct constructions seems to be difficult to find. However, further results in universal algebra provide a concrete reason why these direct constructions are not straightforward.

We will cover the highlights of these results relating to algebraic free objects, including a connection with the *Pierce-Birkhoff conjecture*; a long-standing open problem in real algebraic geometry. Beyond algebra, we also outline a universal construction of analytic free objects using an associated algebraic free object as a 'foundation'. Results on classification of analytic free objects and questions of injectivity of the generator map for both algebraic and analytic free objects will also be discussed.

𝕃-valued integration

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SAMS Classification: 11

We develop integration theory for functions taking values in a Dedekind complete unital f-algebra Ł with respect to Ł-valued measures. We then discuss completeness results of Ł-valued L^p-spaces.

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Local nets of unbounded operator algebras

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

One approach to a rigorous mathematical foundation to relativistic quantum field theory is to make use of local nets of von Neumann algebras, where, for a given region of spacetime, self-adjoint bounded linear operators in a von Neumann algebra are used as observables in the given region of spacetime (we note that the von Neumann algebra of observables generally depends on the region of spacetime under consideration). In reality, the observables are unbounded linear operators on a Hilbert space, so that one would have to consider local nets of unbounded operator algebras in which the closure of the operators are affiliated to a von Neumann algebra (in general, the von Neumann algebra depends on the unbounded operator algebra within the local net under consideration).

There exist various types of unbounded operator algebras in the literature, including GW*-algebras, which are generalizations of von Neumann algebras.

The purpose of this talk is to define local nets of GW*-algebras in a suitable manner, and to give some of their basic properties. For this, one must take into account that there are various ways in which to extend the notion of a commutant of a set of bounded linear operators, to those of unbounded linear operators, and then finding the suitable extension of the concept to physical applications. Connections to Wightman theory will also be discussed, which forms a significant component of the talk.

Totally ordered vector lattices and Artinian and Noetherian vector lattices

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SAMS Classification: 11

In this talk we will first discuss totally ordered vector lattices (an important example is the hyperreals) and discuss their classification, which involves well-ordered sets. Next, inspired by the success of the following notions in ring theory, we turn to the structure of Artinian and Noetherian vector lattices. In our efforts to classify those we turn to some similar ideas as above: we consider well-founded sets, and the important tools of well-founded induction and recursion.

Asymptotics of the eigenvalues of a fourth-order operator with parameter dependent boundary conditions

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SAMS Special Session: Functional Analysis and Operator Theory
SAMS Classification: 11

An eigenvalue problem for a regular fourth order ordinary differential equation is considered, where one of the boundary conditions linearly depends upon the eigenvalue parameter. We investigate the asymptotic expansion of the eigenvalues.

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6 Special Session: Lie symmetries and other approaches in solving nonlinear differential equations

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Analysis of equations arising in radiating stars

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: 5, 10, 20

Radiating stars have attracted much interest over the years. The pioneering work of Santos (1) ensured that the boundary condition at the interface of the interior and exterior of the star could be analysed. Various shear-free and shearing models have been developed over the years. Initially, *ad hoc* approaches were adopted to solve this important equation with varying degrees of success. More recently, the group invariant approach (2,3) as well as the asymptotic approach (4) have been utilised to great effect. The application of Lie symmetries to the underdetermined partial differential equation that arises has revealed interesting insights (5). In the case of ordinary differential equations, the phase plane approach has also been useful (6). We will discuss how these methods have been utilised in recent analyses.

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Approximation of numerical solutions for a chaotic pendulum with parametric excitation

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: 5

This presentation highlights the importance of accuracy checking on built-in algorithms for various mathematical software packages. The built-in algorithms used in this presentation includes Runge-Kutta method of the fourth order(RK4), Adams method, Backward differential formula, AdamsBDF, Radau, Bulstoer, StiffR, Stiffb methods. Pendulum with parametric excitation term is used as case study for chaotic system. The numerical results are presented in both tabular and graphical forms.

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A study of a generalized Zakharov–Kuznetsov equation with dual power-law nonlinearities

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: Analysis of PDEs

In this talk, we employ Lie symmetry analysis [1,2] to study a generalized Zakharov–Kuznetsov equation with dual power-law nonlinearities, which arises in several areas of engineering and nonlinear science. The Lie point symmetries admitted by the equation are systematically derived and subsequently used to reduce the partial differential equation to a set of ordinary differential equations. These reduced equations are then solved by invoking different analytical techniques to construct several classes of exact solutions. Moreover, we derive the conserved vectors by employing both the multiplier method and Noether's theorem in conjunction with the Helmholtz conditions, utilizing the homotopy formula. The obtained results contribute to a deeper understanding of the symmetry structure and conservation properties of nonlinear dispersive wave models with complex nonlinearities.

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Pressure-Driven Flow Analysis in a 3D Filtration System

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: Analysis of PDEs

Filtration systems are crucial in industrial and biomedical processes, where pressure gradients primarily drive fluid transport through porous media. This study presents an analytical investigation of three-dimensional filtration flow under both linear and oscillatory pressure conditions to advance understanding of such systems. The dimensionless continuity and Navier-Stokes equations are solved using Lie symmetry analysis to derive closed-form solutions representing the velocity and pressure fields inside the filter chamber. Thereafter, the obtained solutions are used to analyse momentum and pressure variation inside the filter chamber. The analysis reveals that linear pressure gradients produce stable flow patterns, whereas oscillatory pressure gradients enhance clogging during filtration. These results offer valuable theoretical insights for maximising outflow while enhancing filtration performance.

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Application of Lie symmetry analysis to solve the Sharma-Tasso-Olver equation

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This study investigates the Sharma-Tasso-Olver (STO) equation, a higher-order nonlinear partial differential equation (PDE) that arises in turbulence modelling and shock wave theory. Using Lie symmetry analysis, we derive symmetries that reduce the PDE to an ordinary differential equation (ODE), yielding new insights into its behaviour and solutions. The resulting reduced systems are examined for exact group-invariant solutions, and an optimal system of symmetries is constructed to classify these solutions. Additionally, traveling wave solutions are obtained using the tanh method, and conservation laws are derived using the multiplier method. The results underscore the efficacy of Lie symmetry analysis in simplifying and solving complex nonlinear PDEs, demonstrating its value in both analytical and computational contexts.

Keywords: Lie symmetry, Sharma-Tasso-Olver equation, Conservation laws.

Analysis of Pollutant Diffusion

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: Analysis of PDEs

Pollutant spillages in soil pose a significant risk to the environment, including ground-surface, groundwater, and surface water systems. Quantifying how far contaminants may propagate into the soil and along the ground surface is essential. To understand such dynamics, the study seeks to model and analyse a system of equations representing momentum, heat, and contaminant transport within the ground surface. The model of interest integrates the continuity, Navier-Stokes, and energy equations along with the advection-diffusion-reaction transport equation to study how momentum and energy variation affect the dynamics of pollutant diffusion. Lie symmetry analysis and the perturbation method are used to find solutions representing momentum, energy, and concentration variation as the contaminant propagates. The study seeks to show a correlation between the model and the physical scenario the model represents. Thus, predicts the propagation of the contaminant to understand how the contaminant propagates on different surfaces.

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Solving Linear Reaction-Diffusion Equations Using Lie Symmetry Methods

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: 3

Linear reaction-diffusion equations play a central role in modelling processes such as heat conduction, chemical diffusion, and signal propagation in various scientific fields. Despite their linearity, these equations often exhibit rich dynamical behavior that requires effective analytical tools for their solution. In this research, we apply Lie symmetry methods to obtain exact solutions of linear reaction-diffusion equations. The approach involves identifying the symmetries admitted by the equations, reducing the partial differential equations (PDEs) to ordinary differential equations (ODEs) through similarity transformations, and solving the reduced equations. The Lie symmetry method not only streamlines the solution process but also reveals invariant properties of the equations. This study highlights the effectiveness of symmetry analysis in simplifying and solving linear reaction-diffusion models, offering insights that are valuable in both theoretical investigations and practical applications.

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Lie point symmetry analysis of momentumless thermal wake

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: 3, 5

In this talk, we present a Lie point symmetry analysis of the turbulent momentumless thermal wake flow and provide numerical results. We use the Prandlt mixing length hypothesis to model turbulence brought about by momentum and thermal mixing. The Lie point symmetry analysis includes obtaining conserved vectors of the governing partial differential equations and their associated Lie point symmetries. The governing partial differential equations are nonlinear and coupled. Using the associated Lie point symmetries to reduce the governing partial differential equations to ordinary differential equations (ODEs) guarantees that the ODEs can be integrated at least once by Sjöberg theorem. We employ a shooting method that uses conserved quantities as targets to solve the resulting ODEs.

A pre-existing fluid-driven permeable fracture with Darcy flow

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This study examines the propagation of a pre-existing fluid-driven fracture in a permeable rock. Incompressible laminar Newtonian fluid drives the fracture which experiences fluid loss through the fracture interface into the surrounding rock matrix. Because the Carter's model derived from Darcy law has its many flaws, a new model for the fluid loss relating the leak-off depth to the net fluid pressure in the fracture is employed in this work. The elasticity of the rock is modelled using the Khristianovic-Geertsma-de Klerk (KGD) model. Starting out with lubrication equations, a system of partial integro-differential equations relating the width of the fracture to the net pressure and the leak-off depth is derived. Similarity solutions derived for the fracture half-width, net pressure, and depth of leak-off are used to reduce the system of partial integro-differential equations to a system of ordinary integro-differential equations. Numerical results are obtained for the fracture length, fracture half-width, leak-off depth and the net fluid pressure.

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Solutions and conserved vectors of the 1D KdV equation with nonlinearity power

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SAMS Special Session: Lie symmeteries and other approaches to solving nonlinear differential equations
SAMS Classification: 3, 5

The one-dimensional Korteweg-de Vries (KdV) equation is a model which arises in mathematical sciences. In this talk we explore this model with the use of symmetry analysis[1] technique along with some special techniques such as the direct integration procedure and explicit power series method to produce different closed-form solutions. Additionally, conservation laws of this equation are presented via two differential approaches, namely the multiplier method[2] and Noether's theorem[3]. The conservation laws obtained contains conservation of momentum and energy.

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On the exact solutions of the (1+1)-dissipative Westervelt equation

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SAMS Special Session: Lie symmetries symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: 3

The Westervelt equation plays a crucial role in modeling various physical waves phenomena, in particular sound propagation and its effects on nonlinear media. In this study, we conduct a Lie symmetry analysis of the $(1+1)-$dimensional dissipative Westervelt equation, reducing it to an ordinary differential equations via the optimal system, each yielding invariant solutions. Furthermore, we use sub-equation function method to find additional solutions. The wave profiles of these solutions are visualized through two-dimensional and three-dimensional plots.

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From Single (Symmetry) Reduction via the Optimal System to Double Reduction via Conservation Laws for physically interesting Partial Differential Equations

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification: Analysis of PDEs

In this talk, we begin with symmetry classification [1] for a class of nonlinear PDEs using the Lie symmetry method and discuss the fundamental simplifications of variables that arise from the Lie invariance criterion. Following this, we discuss the construction of the optimal system of subalgebras to identify unique group-invariant solutions. It is well known that a double reduction [2] occurs whenever the starting nonlinear PDE possesses a conservation law that is invariant with respect to the symmetry. In the end, a summary of the multi-reduction method will be presented for obtaining invariant solutions of physically interesting PDEs.

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Symmetry Versus Integrability: A Comparative Analysis of Solution Methods for Dissipative Nonlinear Partial Differential Equations

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SAMS Special Session: Lie symmetry analysis and other approaches in solving nonlinear differential equations
SAMS Classification: 20

This article systematically investigates the relationship between symmetry reductions, such as Lie point symmetries and invariant solutions, and modern integrability techniques for solving dissipative nonlinear partial differential equations (PDEs). Focusing on the prototypical Burgers equation, we examine how Cole-Hopf transformations interact with symmetry-derived solutions, including scaling-invariant shocks, and quantify the discrepancies between viscosity-dominated dynamics and symmetry-constrained behaviour. A central aim of this study is to analyse the alignment or divergence of symmetry reductions (e.g., travelling wave solutions) with integrability-based approaches, while also assessing whether symmetry-invariant solutions can be recovered through Cole-Hopf linearisation. By doing so, this work establishes a systematic classification of PDEs where symmetry methods and integrability techniques either intersect or diverge, while also deriving new analytical solutions for nonlinear PDEs through hybrid methodologies. Ultimately, this research bridges geometric symmetry analysis and analytical integrability, offering a unified framework for addressing nonlinear PDEs. The findings hold potential implications for computational physics, fluid dynamics, and soliton theory, providing deeper insights into the interplay between symmetry and integrability in dissipative systems.

A Robust Direct Block Hybrid Method for Solving Nonlinear Second-Order Differential Equations

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SAMS Special Session: Lie symmetries and other approaches in solving nonlinear differential equations
SAMS Classification:

This paper introduces a robust Block Hybrid Method (BHM) for the direct numerical integration of second-order nonlinear ordinary differential equations (ODEs), with a particular focus on the generalized Van der Pol and Duffing equations. By avoiding the common practice of reducing the problem to a larger first-order system, our method minimizes computational overhead and potential error accumulation. The one-step, self-starting nature of the block formulation provides solutions at multiple grid points simultaneously, enhancing computational efficiency over traditional step-by-step methods. We first establish a priori bounds for the solutions under general conditions to ensure theoretical soundness. The method's efficacy is then validated against several well-known nonlinear problems, demonstrating its high order of accuracy and stability. The results establish the BHM as a highly competitive and reliable alternative for obtaining accurate solutions to second-order initial value problems in applied science and engineering.

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7 Special Session: Mathematics of Gravity

Sessions organisers: S Hansraj (hansrajs@ukzn.ac.za)

Integrability and Well-Posedness Issues in Relativistic and Modified Gravity Field Equations

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SAMS Special Session: Mathematics of Gravity
SAMS Classification:

The formulation of gravitational theories beyond General Relativity (GR) introduces higher-order field equations, additional geometric degrees of freedom, and non-standard constraint structures. These features raise fundamental questions about integrability, consistency, and the well-posedness of the resulting dynamical systems. This talk examines the mathematical foundations of relativistic and modified-gravity field equations with a particular focus on constraint propagation and the conditions required for consistent evolution. Using representative examples from higher-order metric theories, the talk draws on covariant methods to clarify the interplay between constraint equations and dynamical equations, showing where integrability conditions enforce strong restrictions and where modified geometry relaxes classical results. The aim is to provide a unified perspective on the mathematical viability of extended gravitational models and to outline the implications for cosmological modelling and numerical implementation.

Resonant pacemakers heartthrob of Sgr A* emission

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SAMS Special Session: Mathematics of Gravity
SAMS Classification: 10,20,21

This talk explores the most likely timescales and locations that will be observed if a small perturbation is applied to the Kerr metric. Brandon Carter showed that the geodesic equations in the Kerr metric form an integrable system of equations. If a weak non-linear perturbation is applied to the integrable system, the Kolmogorov-Arnold-Moser (KAM) Theorem implies that the largest, possibly non-smooth, departures in the dynamics, will occur near low order resonant orbits. We quantify all locations in the Kerr spacetime where resonant orbits occur and calculate the characteristic timescales associated with the resonant regions. We show that several observed temporal periodicities in the X-ray and near infrared flare emission from Sgr A* can be associated with prograde resonant structures around a Kerr black hole with spin $a = 0.5$. Resonant structures can be stabilizing; a classical example is the formation of Saturn's rings due to gas being captured in regions where resonances with the orbiting moons and the internal vibrations of the planet occur. We show that the locations of orbiting hotspots, around Sgr A*, resolved by the Very Large Telescope and the Event Horizon Telescope can be associated with resonant structures. This fact provides a mechanism that explains both the formation and continued stability of these structures at their observed locations.

Minimal Decoupling Constraints on Geometrically Perturbed Strange Stars from Massive Millisecond Pulsars

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SAMS Special Session: Mathematics of Gravity
SAMS Classification:

We develop an exact analytical framework for describing the internal structure of strange stars within general relativity. To capture more realistic astrophysical conditions, we introduce a smooth spacetime deformation using a sinusoidal perturbation, $g(r) = \sin(\Psi r^2)$, which naturally generates internal anisotropy, expressed as $\Delta = P_t - P_r > 0$. This anisotropy provides additional support against gravitational collapse, enhancing the star's stability. Pulsars—extremely dense, rapidly rotating neutron stars that emit highly regular electromagnetic pulses—serve as excellent laboratories for probing gravity in its strongest regimes. Their extreme compactness allows us to test how general relativity behaves under intense gravitational fields. By applying Israel-Darmois junction conditions, we seamlessly match the star's interior geometry to the Schwarzschild exterior, deriving the model constants in terms of the geometric parameters β and Ψ. These parameters quantify the effects of subtle spacetime distortions on the star's equilibrium and structural properties. Our analysis shows that increasing β from 0 to 0.003 (with Ψ fixed) leads to a significant increase in the maximum mass, reaching $2.28^{+0.10}_{-0.09} M_\odot$, while the radius grows to $11.57^{+0.87}_{-0.10}$ km before the onset of instability. Comparable trends are observed when Ψ is varied up to ~ 0.03 at fixed β. The resulting mass–radius relations are consistent with observations of massive pulsars such as PSR J0740+6620 ($2.08^{+0.07}_{-0.07} M_\odot$), PSR J1810+1744 ($2.13^{+0.04}_{-0.04} M_\odot$), PSR J1959+2048 ($2.18^{+0.09}_{-0.09} M_\odot$), and PSR J2215+5135 ($2.28^{+0.10}_{-0.09} M_\odot$), supporting the model's physical reliability. Throughout the star, energy density and pressures remain smooth and positive, while sound speeds stay subluminal. The adiabatic index Γ consistently exceeds the critical 4/3 threshold, confirming dynamical stability. Interestingly, simultaneous increases in β and Ψ produce subtle oscillatory variations in mass and radius—a "breathing" effect—highlighting how even minimal spacetime distortions can influence the long-term evolution of compact stars. Overall,

this model provides mass–radius predictions in close agreement with current astrophysical measurements, offering a robust and physically consistent framework to explore how strange stars respond to subtle spacetime deformations and environmental perturbations.

Curvature singularities and mass gap in modified gravity

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SAMS Special Session: Mathematics of Gravity
SAMS Classification: 20

The most widely accepted theory of gravity is Einstein's theory of general relativity (GR). While it has passed many tests, a critical shortcoming is its inability to explain the observationally confirmed accelerated expansion of the Universe without referencing dark matter or including the cosmological constant. This necessitates extensions to the theory, leading to modified gravity theories, like the Einstein-Gauss-Bonnet (EGB) framework, which include higher curvature terms. In this talk, I will provide an overview and substantiation of the theory. Then I will explore two critical features of modified gravitational systems: curvature singularities and mass gaps. Investigations into these features are central to our understanding of the structure and stability of compact objects in higher-dimensional gravity settings and for comparisons to classical GR. We first provide a geometrical explanation for the mass gap phenomenon in 5D EGB black holes via spacetime decomposition (1). We show in the case of self-similar radiation collapse, the central singularity is not a sink for timelike geodesics and is extendable. Thereafter, we consider N-dimensional cosmology within the perfect fluid EGB framework, investigating the nature of singularities and the evolution of the Universe under nonstandard conditions (2). These studies clearly demonstrate how the Gauss-Bonnet invariant affects the nature of the final state of collapse in this modified theory.

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Massive gravity: massive mathematical headaches

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SAMS Special Session: Mathematics of Gravity
SAMS Classification: Mathematical Physics (20)

Einstein's general relativity (GR) is still the leading theory of gravitational interactions. However strong reasons exist to believe that it is in need of extension. Firstly it is not renormalizable like its counterpart quantum theory and relies on exotic matter fields to explain the observed accelerated expansion of the universe. To modify the theory requires careful mathematical considerations. GR is a spin-2 theory (in quantum language) and is diffeomorphism invariant, must satisfy the Bianchi identities and generate up to second order differential equations describing the field. Fierz and Pauli originated the idea that gravity could propagate through a massive hypothetical particle known as a graviton. Their linearised proposal failed to recover GR. Allowing nonlinearity was useful to regain GR however extra degrees of freedom arose known as Boulware-Deser ghosts. Several approaches were constructed to eliminate the ghosts and recently the DRGT (de Rham-Gabadadze-Tolley) construction resolved the ghost problem with a novel definition of the concept of the square root of a tensor and a reference metric. On account of its mathematical complexity the study of astrophysical phenomena was a no-go area. Recently Vegh introduced a suitable reference metric however the construction sacrificed diffeomorphism invariance, translational symmetry and Lorentz invariance. We use such a reference metric to probe the impact of massive gravitons on stellar structure. In particluar we analyse the equation of state, stability criteria and energy conditions. Several mathematical configurations such as algebraically special spacetimes such as conformal flatness, metrics admitting groups of conformal motions as well as the embedding problem are studied.

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Compact stars in regularized $4D$ Einstein-Gauss-Bonnet gravity with non-minimal coupling

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SAMS Special Session: Mathematics of Gravity
SAMS Classification: Mathematical Physics

We construct a new exact anisotropic compact-star solution in regularized four-dimensional Einstein-Gauss-Bonnet ($4D$ EGB) gravity with a non-minimal scalar-GB coupling $\xi(\phi)\mathcal{G}$. Utilising a Finch-Skea ansatz for the metric potentials, we consider a massless scalar field and an exponential coupling $\xi(\phi) = \xi_0 e^{\kappa\phi}$ The field equations in the $D \to 4$ regularized limit are solved in closed form, yielding explicit expressions for the density, pressures, the equation-of-state indicators, directed sound speeds and adiabatic indices together with the mass function and compactness. We show that the model admits the Einstein limit when the effective GB parameter is set to zero, and we implement exterior matching to the $4D$ EGB Boulware-Deser–type solution to fix the necessary integration constants via continuity and the vanishing radial pressure surface condition. For a carefully selected set of model

parameters generated through empirical fine-tuning we obtain a monotonically decreasing density and pressures, satisfaction of the energy conditions, causal sound speeds, Herrera cracking compliance, and adiabatical stability throughout the interior. The compactness remains sub-Buchdahl while the GB coupling tends to increase the stellar radius for fixed central conditions. These results provide a transparent exact benchmark for probing GB–scalar effects in realistic stellar interiors.

Dynamical System Analysis of Interacting Dark Energy Models

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SAMS Special Session: Mathematics of Gravity
SAMS Classification: 10

Interacting dark energy (IDE) models introduce a non-gravitational exchange of energy between dark matter and dark energy, often leading to background evolution equations that have no analytical solutions. In this work, we study the model with interaction $Q = 3H\delta\rho_{\mathrm{de}}$ using a dynamical systems approach, with emphasis on the mathematical structure of the phase space.

We rewrite the evolution equations for radiation Ω_{r}, baryons Ω_{bm}, dark matter Ω_{dm} and dark energy Ω_{de} as an autonomous system, and analyse its behaviour through its critical points, invariant submanifolds and stability properties. The system contains three equilibrium points: a radiation-dominated source, a matter-dominated manifold of saddle points, and a dark-sector hybrid attractor. By identifying the invariant boundaries associated with positive energy densities ($\Omega_i \geq 0$), we show that physically viable trajectories are confined to a triangular region in the $(\Omega_{\mathrm{dm}}, \Omega_{\mathrm{de}})$ plane. This leads to explicit constraints on the interaction parameter δ, ensuring non-negative energy densities throughout the evolution.

These conditions match those previously obtained from direct analytical solutions, but here arise solely from the geometry and stability of the dynamical system. The framework developed can be applied to a wide class of interacting dark-sector models where analytical solutions are unavailable.

Finally, we briefly discuss how the obtained critical points may be used to determine the final fate of the universe.

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Imaging resonant structures around a Schwarzschild black hole

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SAMS Special Session: Mathematics of Gravity
SAMS Classification: 10, 20, 21

We present the first gravitationally lensed image of the location of resonant structures embedded in an equatorial disc around a Schwarzschild black hole. Unlike the current imaging methods, which rely heavily on computationally expensive numerical methods, our imaging technique is mostly analytical. We used methods like Carlson's integrals to determine the geodesic constants that connect an observer and emitter through any nth-order light ray. This rigorous method permits very rapid image generation. Several images that explore the impact of varying the observer's location relative to the disc are generated. We further explore the characteristic temporal emission features of a hotspot on a resonant orbit.

8 General Session

On multiphase drift-flux gas flows through a networked domain

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SAMS Special Session: General Session
SAMS Classification: 3, 23

Networks of one-dimensional flows offer a rich framework for modelling dynamical processes across diverse phenomena – from gas and water transport to traffic and circulatory systems. These models combine the analytical complexity of partial differential equations (PDEs) with the topological structure of graphs. This talk explores some developments in the mathematical analysis, numerical approximation, in the particular case of multiphase drift-flux gas flows. The work includes well-posedness of PDEs on networks, coupling conditions at junctions, and numerical consideration with computational examples.

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Numerical analysis of pattern formation in reaction-diffusion systems

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SAMS Special Session: General Session
SAMS Classification: 3

The presentation will explore the numerical analysis of pattern formation in reaction-diffusion systems, focusing on the Allen-Cahn and Cahn-Hilliard equations. The study compares different numerical approaches, including the finite difference method coupled with the pseudospectral method (FD-PSM) and a hybrid block method coupled with the pseudospectral method (HB-PSM). Emphasis is placed on how these techniques capture the dynamics and symmetry of emerging patterns, as well as the efficiency and accuracy of the schemes.

Involute Origami Flashers

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SAMS Special Session: General Session
SAMS Classification: 5,9,20,21

I construct and analyze a new curved fold origami flasher crease pattern based on involute curves. Origami flashers are often used in satellite missions to package large solar arrays into the fuselage of a rocket before launch. Once in space the array is unfurled so that the maximum collecting area is attained. In the search for extra solar planets an origami flasher was also employed to package the 34m occulter into the 5m rocket fairing.

Mathematically origami is the study of how a given set of curves or creases drawn on a 2D sheet of paper can be deformed into a 3D structure. The case where the curves are straight lines is well understood, all aforementioned flasher examples fall into this category. Curved fold origami on the other hand still has a number of open geometric questions and few curved flasher designs are known.

An involute is the path traced by a point on a string as it is unwound from a base curve. These curves are often used in mechanics for designing gear teeth. The particular flasher design presented here is based on the properties of these involute curves. Curved fold origami provides a remarkably fun and intuitive way of introducing geometric concepts such ruled surfaces, intrinsic and extrinsic curvature so often used in courses on general relativity and manifolds. I illustrates these ideas using my new flasher design.

Stable and accurate finite difference approximation of the incompressible thermal boundary layer equations

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SAMS Special Session: General Session
SAMS Classification: 12

In this work we develop and validate a stable and high-order finite difference approximation for the incompressible thermal boundary layer equations using the Summation-By-Parts (SBP) and Simultaneous Approximation Term (SAT) framework. We begin by establishing the theoretical foundations of well-posedness and stability through the energy method. We then extend the analysis to the discrete setting. This is done using the SBP operators to mimick the continuous integration-by-parts property, and SAT techniques to weakly enforce boundary conditions to ensure discrete energy stability. Next, we verify high-order accuracy using the method of manufactured solution. The framework is subsequently applied to the incompressible Navier–Stokes equations with thermal energy equation included, modeling laminar flow over an isothermal flat plate. The computed numerical results are benchmarked against the *Blasius–Pohlhausen* similarity solution, and demonstrate excellent agreement.

On Generalized Bell polynomials

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SAMS Special Session: General Session
SAMS Classification: 06

Generalized Bell polynomials can be defined using the notion of generalized
exponential polynomials. Several important results were developed by alge-
braic manipulations of generalized exponential polynomials . In this talk,
we present a combinatorial interpretation of some of these results using the
notion of 'unfair' distributions.

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Numerical Solution of a Class of Variable order differential equations

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SAMS Special Session: General Session
SAMS Classification: 5, 9

The history of fractional calculus (FC) goes back more than 300 years. In this work, we solve a class of variable order differential equations by using operational matrices based on Hosoya polynomials. We obtain operational matrices for Variable/fractional order derivative and substitute into main equations and convert it to system of functional equations. After that we use collocations points to convert it to a system of algebraic equations. Finally by solving the algebraic system we obtain the numerical solution of governing equations.

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A continuous-time Markov Chain model of substance abuse–driven criminality: Exploring the stochastic dynamics of addiction and crime

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SAMS Special Session: *General Session*
SAMS Classification: *10*

In this work, we design and investigate a model based on a Continuous-Time Markov Chain (CTMC) to construct a framework for examining the dynamics of drug abuse and its criminogenic consequences. The model is structured around seven interacting compartments representing different population groups, including susceptible, light, and heavy drug users, individuals in rehabilitation, criminals, incarcerated individuals, and those recovered. By assuming that the susceptible population remains approximately constant during the early stages, we apply a multitype branching process framework to derive a system of offspring probability generating functions (PGFs) and ultimately determine the spectral radius of our model. These functions enable the computation of the extinction probabilities for each initiated compartment. The criticality of the model is evaluated based on the leading eigenvalue of the mean matrix $\mathcal{M}$. If $\rho(\mathcal{M}) \leq 1$, the drug–crime process dies out with complete certainty. However, when the spectral radius $\rho(\mathcal{M}) > 1$, there is a non-trivial fixed point $(0.1)^6$, indicating a positive probability of sustained narcocriminal activity. The numerical solutions of the CTMC model align closely with those of the deterministic model. Although the drug-criminogenic growth number exceeds one, our findings indicate that when a single infected individual is introduced into any of the six infected compartments, the drug-crime epidemic is more likely to die out. In contrast, if all infected compartments contain at least one infected individual at the outset, the likelihood of an epidemic outbreak significantly increases.

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Orthogonality of Quasi-Christoffel Polynomials of Order One: Jacobi and Laguerre Type

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SAMS Special Session: General Session
SAMS Classification: 5

In this talk, I will discuss the linear combinations of two consecutive polynomials generated through spectral transformations, focusing mainly on the Christoffel transformation. These are known as quasi-spectral polynomials of order one. I will particularly address how orthogonality, which is generally lost in such combinations, can be restored through the construction of quasi-Christoffel polynomials of order one. Explicit coefficient expressions for the Jacobi and Laguerre cases are obtained, and it is observed that one zero lies on the boundary of the support of the measure.

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Impact of Control Measures on Predator-Prey Interactions with Infected Prey: Analysis and Optimal Solutions

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SAMS Special Session: General Session
SAMS Classification: 25

This study explores the impact of control measures on predator–prey interactions in ecosystems where disease affects prey populations. A mathematical model is developed to describe the dynamics of susceptible prey, infected prey, and predators, incorporating spatial diffusion and convection to capture population movement. Infected prey are assumed to be more vulnerable to predation, reflecting realistic ecological behavior.

Two control strategies are proposed: treatment of infected prey to reduce disease susceptibility and predator speed reduction to regulate predation pressure. The objective is to minimize a cost functional that balances ecological stability and management expenses. The existence of an optimal control strategy is established using the direct method in the calculus of variations, ensuring coercivity, convexity, and lower semicontinuity of the objective functional.

Numerical simulations show that increasing treatment rates significantly reduces infection prevalence and stabilizes prey populations, while limiting predator speed supports the persistence of susceptible prey. Sensitivity analysis identifies the disease transmission rate and natural death rate of infected prey as key parameters driving system behavior. The results demonstrate how optimal control theory can effectively guide ecological management and conservation in predator–prey systems influenced by disease dynamics.

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A finite element solver for the Poisson equation with a matrix-valued coefficient

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SAMS Special Session: General Session
SAMS Classification: 12

In this work we consider the Poisson equation subject to a Dirichlet boundary condition. The problem is transformed to a mixed formulation on which a Galerkin finite element method is applied. This results in a linear system where the system matrix is symmetric. A solver which is known to work well in this case is the Conjugate gradient method. Computer implementation of the overall numerical solver makes use of `deal.II`, a `C++` library that generate finite element codes for solving a wide range of differential problems. A test case for the problem considered here is used to demonstrate the computationally efficiency of the numerical solver.

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Double-diffusive Convection with gravity and temperature modulation in a rotating fluid using the non-traditional approximation

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SAMS Special Session: General Session
SAMS Calssification: Mathematical physics

This study investigates double-diffusive convection in a rotating stratified oceanic fluid layer subjected to simultaneous gravity and temperature modulation. The analysis is performed under the non-traditional approximation of the Coriolis effect, thereby incorporating both the vertical and horizontal components of Earth's rotation, which are essential for accurately capturing equatorial and mid-latitude oceanic dynamics. Temperature modulation is introduced via the Garlakin approach and analyzed using Floquet theory to account for the periodic thermal forcing. Gravity modulation is incorporated through the method of normal modes, enabling the assessment of vertical oscillatory effects on buoyancy-driven flow instabilities. The governing hydrodynamic and thermodynamic equations are linearized to perform a linear stability analysis and determine the conditions for the onset of convection in the presence of salinity gradients and rotational influences. Our results highlight the combined role of modulation frequencies, diffusivity ratios, and rotational parameters in shaping the stability boundaries of double-diffusive systems. The findings provide insight into oceanic mixing processes influenced by tidal forcing and planetary rotation, contributing to improved understanding of large-scale ocean circulation and stratification dynamics.

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Additional food provided to intermediate predator in a prey, predator, super-predator model

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SAMS Special Session: General Session
SAMS Classification: 10

In this study, the dynamics of a three-species system involving prey, predator and super-predator in the presence of additional food provided to the predator are studied. The predator and super-predator consume the common prey. Due to large competition from the dominant super-predator, the predator faces extinction. Provision of additional food to the predator is considered as a means to resuscitate the low predator population. Mathematical analysis has been employed to derive conditions for the stability of solutions. The results indicate that additional food plays a pivotal role in rescuing the predator species from extinction.

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Non-similar solutions for Darcy-Forchheimer flow with multiple dissipative mechanisms

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SAMS Special Session: General Session
SAMS Classification: Analysis of PDEs

The Darcy-Forchheimer flow of an electrically conducting, dissipative fluid over a stretching flat sheet is investigated, considering three distinct dissipative mechanisms: viscous, ohmic, and porous dissipation. The governing equations are modelled to analyze the flow and thermal characteristics within the boundary layer region using suitable assumptions. A novel transformation is introduced wherein the Forchheimer number varies along the streamwise direction. This transformation yields a set of non-similar partial differential equations from the governing boundary layer equations. The numerical solutions of these nonlinear equations are obtained using an efficient computational approach, namely the overlapping multi-domain spectral collocation scheme. The influence of various flow parameters on different flow characteristics and relevant quantities is presented graphically and discussed in detail. Important findings include that the velocity profile decreases with increasing magnetic parameter and Forchheimer number but increases with a higher porosity parameter. Temperature gradients near the wall intensify at higher Eckert numbers and magnetic parameters, while thermal boundary layer thickens with a higher Forchheimer number. These findings are significant for optimizing engineering systems involving porous materials, heat exchangers, and systems influenced by magnetic fields and inertial effects.

Criminodynamics: Modelling Crime in Communities and Correctional Facilities.

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SAMS Special Session: General Session
SAMS Classification: 10

Crime continues to pose a significant threat to socio-economic development and public safety, particularly in South African urban and peri-urban areas where social inequality, unemployment, and limited access to rehabilitation services persist, (1). The model is informed by publicly available crime and correctional services data from high-incidence areas in Gauteng Province, including Johannesburg Central, Hillbrow, and parts of Tshwane, where crime density and recidivism rates are notably high, (2). This study presents a differential equation model similar to the one used in (3) to simulate and analyze the dynamics of criminal activities within communities and correctional facilities. The model considers interactions between individuals in the community, including both law-abiding citizens and criminals, as well as the effects of incarceration and rehabilitation in correctional facilities. The main focus is on identifying steady states of criminal behaviour under various scenarios, including different levels of law enforcement, rehabilitation effectiveness, and social interactions. Mathematical analysis techniques, such as stability analysis and bifurcation theory, are employed to characterize the equilibrium points and their stability, building upon findings from (4). Furthermore, numerical simulations are conducted to validate the theoretical findings and explore the transient dynamics of the model. These simulations provide insights aligning with (5) into how parameter changes affect crime rates over time and the effectiveness of intervention strategies. Overall, this research contributes to understanding the complex dynamics of crime within communities and the impact of correctional services, offering a quantitative framework for policy-makers and law enforcement agencies to develop more effective crime prevention and rehabilitation strategies.

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Summation-By-Parts operators for general function spaces: optimal quadrature nodes and weights

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SAMS Special Session: General Session
SAMS Classification: 12

The augmentation of the Summation-By-Parts (SBP) formulations with the implementation of weak boundary conditions via the Simultaneous Approximation Term (SAT) technique guarantees semi-discrete stability for linear well-posed initial boundary value problems. This framework was initially developed to achieve stability for high-order finite difference approximations, but it has since been extended to encompass nearly all existing spatial discretizations, including finite volume, spectral element, and Galerkin methods.

Current SBP operators are designed to be exact for polynomials up to a certain degree. In this presentation, we explore a generalized function spaces that includes non-polynomial functions and construct the corresponding SBP operators with optimal quadrature nodes and weights. We demonstrate their accuracy and efficiency across different function spaces by solving initial boundary value problems.

Using Bernoulli Operational Matrix To Solve a Pancreatic Cancer Model with Caputo Fractional Operator

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SAMS Special Session: General Session
SAMS Classification:

This research adopts the eco-epidemiological model of Thakkar et al. [1] as a foundation. The model employs Caputo fractional derivatives and describes interactions between pancreatic cancer cells, pancreatic stellate cells, effector immune cells, and cytokines. To solve this model, we present a numerical method based on the operational matrix of Bernoulli polynomials. By expressing the solution as an expansion of Bernoulli polynomials and combining it with a collocation approach, the problem is transformed into a system of nonlinear algebraic equations. Numerical simulations demonstrate the accuracy and effectiveness of the proposed method. Furthermore, we investigate how changes in effector immune cell levels influence the system dynamics.

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Solving higher order Korte De Vries(KdV) equations using multi-domain spectral collocation method

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Solving KdV equations has been Long-standing challenge particularly for large computational domains. This study introduces the multi-domain spectral collocation method for solving higher order Korteweg De Fries (KdV) equations defined on large time frames. The KdV equations are firstly altered to a linearized form of iterative scheme using quasi-linearization method. The time domain is decomposed into non-overlapping sub-intervals, while on the contrary space domain is disintegrated into overlapping sub-intervals. The solutions at each time sub-interval are computed independently, where continuity condition is applied to produce initial conditions for the subsequent subintervals, whereas the solutions in the space interval are to be determined simultaneously across overlapping sub-intervals. The solutions are correlated with the exact solutions to manifest accuracy. Furthermore, the efficacy, stability and accuracy of the method are demonstrated by presenting computational error analysis, condition numbers and the computational time for the solution of KdV equations.The method has proven to be a reliable numerical tool, capable of accurately capturing the intricate structures typical of KdV-type models, such as solitons and complex wave interactions. The method also demonstrated high accuracy, with numerical solutions closely matching available exact solutions, yielding small absolute errors.

Keywords: KdV equation, Quasi linearization method ,Multidomain spectral collocation method, bi-variate lagrange interpolation

Modelling the dark side of prison dynamics: Quantifying the criminogenic effects of petty crime offenders' incarceration

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SAMS Special Session: General Session
SAMS Classification: 10

Crime remains a persistent socio-economic challenge in South Africa, where the incarceration of petty offenders often exacerbates rather than mitigates criminality. Research indicates that custodial sentencing for minor offences can accelerate progression to serious crime due to exposure to hardened offenders and criminogenic prison environments. To explore this dynamic, we developed a compartmental mathematical model using ordinary differential equations to represent transitions between key states: susceptibility to crime, engagement in petty and non-petty offences, sentencing types, rehabilitation, and relapse. Two threshold parameters are introduced: the crime generation number ($\mathcal{R}_0$), representing the average number of new criminal cases generated by one offender, and the carceral-induced criminality ratio (CCR), quantifying the risk of incarceration contributing to increased criminal activity. The model is parameterised using South African crime and prison data and calibrated against observed incarceration trends. Sensitivity analysis identifies sentencing rates, rehabilitation success, and relapse probabilities as critical drivers of long-term dynamics. Simulations reveal that reliance on custodial sentencing sustains high recidivism and incarceration rates. Conversely, prioritising non-custodial sentencing and strengthening rehabilitation efforts significantly reduce both $\mathcal{R}_0$ and CCR. These findings underscore that indiscriminate incarceration of petty offenders perpetuates cycles of criminal behaviour, while targeted, evidence-based interventions offer more sustainable crime reduction strategies. This study provides a quantitative framework for evaluating justice system policies and advocates for reforms that balance deterrence with effective rehabilitation to achieve lasting reductions in criminal activity.

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Numerical Comparison Analysis of Implicit Factorized and Localized Radial Basis Function Methods for Shock Wave Boundary Layer Interaction over a Flat Plate with an Imposed Oblique Shock

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SAMS Special Session: General Session
SAMS Classification: 3

This study provides a rigorous quantitative comparison of two distinct numerical methods, the Implicit Factorized Method (IFM) and the Localized Radial Basis Function (LRBF) method, for simulating shock-wave boundary layer interactions (SWBLI). SWBLI is a critical phenomenon in high-speed aerodynamics, impacting vehicle performance and structural integrity. While general trade-offs are known, a direct quantitative benchmark of their ability to predict critical SWBLI-induced phenomena, such as flow separation, wall shear stress, and thermal loads, has been lacking. We address this gap by simulating an oblique shock interacting with a flat plate boundary layer. Results, benchmarked against established data, show that LRBF methods provide superior predictions of separation bubble length and wall-based quantities. The IFM, while faster, suffers from significant numerical diffusion, which smears key flow features, underestimates wall stresses, and fails to predict flow separation. This work offers crucial guidance for selecting appropriate numerical schemes, advocating for high-fidelity methods like LRBF for accurate aerodynamic and thermal analysis in high-speed flows, especially when detailed boundary layer physics are paramount.

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Mathematical modelling of the tumour treatment with chemotherapy

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SAMS Special Session: General Session
SAMS Classification: 10

In this study, a cancer disease model incorporating dynamics between immune cells and tumour cells was formulated in an attempt to understand the interaction dynamics that govern the complex interplay within the tumour microenvironment. Mathematical analysis has been employed to derive conditions for the boundedness of solutions and to discuss disease thresholds such as the basic reproduction number. Furthermore, the model was extended to include chemotherapy treatment and reformulated as an optimal control problem. Optimal control techniques have been applied to examine the role of chemotherapy in enhancing tumour cell elimination and minimising adverse effects on immune cells and normal cells. The results indicate that a low concentration of the drug leads to a prolonged period for tumour clearance. Conversely, a higher drug concentration resulted in a quicker tumour clearance, albeit with adverse effects on normal and effector cells. The optimal control identifies when to stop the treatment once the tumour clears. Furthermore, optimal control facilitates the regeneration of normal and immune cells beyond treatment, in contrast to a control scenario. The findings highlight the importance of early detection and careful chemotherapy dosage selection for effective and personalised cancer treatment strategies.

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Groups Arising from Affine Subgroups of $Sp_{2n}(2)$

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SAMS Special Session: General Session
SAMS Classification: 15

The symplectic group $Sp_{2n}(2)$ admits an affine maximal subgroup of the form $ASp_n = 2^{2n-1}{:}Sp_{2n-2}(2)$, realized as a split extension of an elementary abelian 2-group $N = 2^{2n-1}$ by $G = Sp_{2n-2}(2)$. Notably, the vector space N and its dual N^* are inequivalent as G-modules over $\mathrm{GF}(2)$, both of dimension $2n-1$. This leads to the existence of a non-isomorphic split extension $\overline{G}_n = N^*{:}Sp_{2n-2}(2) \not\cong N{:}Sp_{2n-2}(2)$. In this work, we demonstrate that $\overline{G}_n \cong \mathrm{Aut}(2^{2n-2}{:}Sp_{2n-2}(2))$ for all $n \geq 3$. Furthermore, we investigate the ordinary irreducible characters of $\overline{G}_n$.

Short-Term Forecasting of the JSE All-Share Index Using Gradient Boosting Machines

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SAMS Special Session: General Session
SAMS Classification:

This study applies Gradient Boosting Machines (GBMs) and principal component regression (PCR) to forecast the closing price of the Johannesburg Stock Exchange (JSE) All-Share Index (ALSI), using daily data from 2009 to 2024, sourced from the Wall Street Journal. The models are evaluated under three training–testing split ratios to assess short-term forecasting performance. Forecast accuracy is assessed using standard error metrics: mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE), and mean absolute scaled error (MASE). Across all test splits, the GBM consistently achieves lower forecast errors than PCR, demonstrating superior predictive accuracy. To validate the significance of this performance difference, the Diebold–Mariano (DM) test is applied, confirming that the forecast errors from the GBM are statistically significantly lower than those of PCR at conventional significance levels. These findings highlight the GBM's strength in capturing nonlinear relationships and complex interactions in financial time series, particularly when using features such as the USD/ZAR exchange rate, oil, platinum, and gold prices, the S&P 500 index, and calendar-based variables like month and day. Future research should consider integrating additional macroeconomic indicators and exploring alternative or hybrid forecasting models to improve robustness and generalisability across different market conditions.

Keywords: all-share index; financial market; Gradient Boosting Machines; Johannesburg Stock Exchange; machine learning; time-series predictions

Entropy Generation in Two-Phase Nanofluid Flow over a Permeable Exponentially Stretching Surface

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SAMS Special Session: *General Session*
SAMS Classification:

Type main body of abstract here (Entropy generation quantifies the irreversibility in thermal-fluid systems and plays a central role in evaluating energy efficiency for advanced applications such as microelectronics cooling, nanofluid-based heat exchangers, and polymer processing. This work focuses on the analysis of entropy generation in a two-phase viscoelastic nanofluid subjected to boundary layer flow over a permeable exponentially stretching surface within a porous medium. The governing system incorporates porous resistance, viscous dissipation, Brownian motion, and thermophoresis, influencing velocity, thermal, and concentration fields. Nonlinear partial differential equations describing mass, momentum, energy, concentration, and entropy transport are reduced to ordinary differential equations through similarity transformations. These equations are reformulated into an initial value problem via the shooting method combined with the Newtonâ€"Raphson approach and numerically solved using the Rungeâ€"Kuttaâ€"Fehlberg algorithm. The results show that increasing permeability and viscoelasticity suppress velocity and skin friction, whereas Brownian diffusion and viscous dissipation intensify both thermal fields and entropy generation. Thermophoretic effects increase nanoparticle concentration while reducing wall heat transfer rates. Entropy production decreases with enhanced permeability but grows sharply with stronger viscous dissipation, consistent with observed Bejan number trends. The findings highlight strategies to optimize nanofluid-based energy and cooling systems by balancing heat transfer enhancement against thermodynamic losses.)

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Seeing Functions Differently: A Pedagogical Approach Using Digital Images as Functions

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SAMS Special Session: General Session
SAMS Classification: 16

The function concept is fundamental to mathematics, yet persistent difficulties in its understanding remain evident from secondary to tertiary levels. This presentation reports on research that explores the potential of digital images as functions to support students' conceptual development of the function concept. In the initial study (1), the Image Functions Intervention (IFI) was theoretically evaluated through the Action-Process-Object-Schema (APOS) framework and empirically tested with undergraduate students. By interpreting photographs as functions $f(x, y)$, where each pixels coordinates map to a unique colour value, the IFI created opportunities for learners to experience the function concept dynamically and visually. Qualitative findings revealed broadened concept images, with students beginning to appreciate the dependence of outputs on inputs, the roles of domain and range, and the existence of non-formulaic functions.

Building on this foundation, the follow-up study (2) transformed the intervention into a practical guide for teachers, offering classroom activities that link mathematical ideas such as transformations, linearity, and function composition - to digital image manipulation. The approach encourages learners to see functions not merely as algebraic rules but as relations governing visual and structural change.

Together, these studies illustrate how integrating digital imagery into mathematics education can bridge formal and intuitive understandings, foster conceptual coherence across representations, and re-energise the teaching of functions in secondary and tertiary classrooms.

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